B. E. CIVIL ENGINEERING
Choice Based Credit System (CBCS) and Outcome Based Education (OBE)
SEMESTER - V

ANALYSIS OF INDETERMINATE STRUCTURES

Course Code	**18CV52**	CIE Marks	40
Teaching Hours/Week(L:T:P)	(3:2:0)	SEE Marks	60
Credits	04	Exam Hours	03

Course Learning Objectives: This course will enable students to

1. Apply knowledge of mathematics and engineering in calculating slope, deflection, bending moment and shear force using slope deflection, moment distribution method and Kani's method.
2. Identify, formulate and solve problems in structural analysis.
3. Analyze structural system and interpret data.
4. use the techniques, such as stiffness and flexibility methods to solve engineering problems
5. communicate effectively in design of structural elements

Module-1

Slope Deflection Method: Introduction, sign convention, development of slope deflection equation, analysis of continuous beams including settlements, Analysis of orthogonal rigid plane frames including sway frames with kinematic indeterminacy≤3.

Module-2

Moment Distribution Method: Introduction, Definition of terms, Development of method, Analysis of continuous beams with support yielding, Analysis of orthogonal rigid plane frames including sway frames with kinematic indeterminacy ≤3.

Module-3

Kani's Method: Introduction, Concept, Relationships between bending momentand deformations, Analysis of continuous beams with and without settlements, Analysis of frames with and without sway.

Module-4

Matrix Method of Analysis (Flexibility Method) : Introduction, Axes and coordinates, Flexibility matrix, Analysis of continuous beams and plane trusses using system approach, Analysis of simple orthogonal rigid frames using system approach with static indeterminacy ≤3.

Module-5

Matrix Method of Analysis (Stiffness Method): Introduction, Stiffness matrix, Analysis of continuous beams and plane trusses using system approach, Analysis of simple orthogonal rigid frames using system approach with kinematic indeterminacy ≤3.

Course Outcomes: After studying this course, students will be able to:

1. Determine the moment in indeterminate beams and frames having variable moment of inertia and subsidence using slope defection method
2. Determine the moment in indeterminate beams and frames of no sway and sway using moment distribution method.
3. Construct the bending moment diagram for beams and frames by Kani's method.
4. Construct the bending moment diagram for beams and frames using flexibility method
5. Analyze the beams and indeterminate frames by system stiffness method.

Question paper pattern:

- The question paper will have ten full questions carrying equal marks.
- Each full question will be for 20 marks.
- There will be two full questions (with a maximum of four sub- questions) from each module.
- Each full question will have sub- question covering all the topics under a module.
- The students will have to answer five full questions, selecting one full question from each module.

Textbooks:

1. Hibbeler R C, **" Structural Analysis",** Pearson Publication
2. L S Negi and R S Jangid, **"Structural Analysis",** Tata *McGraw-Hill* Publishing Company Ltd.
3. D S PrakashRao, **"Structural Analysis: A Unified Approach"** , Universities Press
4. K.U. Muthu, H. Narendraetal, **"Indeterminate Structural Analysis",** IK International Publishing Pvt. Ltd.

Reference Books:

1. Reddy C S, **"Basic Structural Analysis",**Tata McGraw-Hill Publishing Company Ltd.
2. Gupta S P, G S Pundit and R Gupta, **"Theory of Structures"**, Vol II, Tata McGraw Hill Publications company Ltd.
3. V N Vazirani and M MRatwani, **"Analysis Of Structures** ", Vol. 2, Khanna Publishers
4. Wang C K, **"Intermediate Structural Analysis",** McGraw Hill, International Students Edition.
5. S.Rajasekaran and G. Sankarasubramanian, **"Computational Structural Mechanics",** PHI Learning Pvt. Ltd.

Development of Slope-Deflection Equation

Let AB, shown in fig(1), be a member of a rigid structure. After loading it undergoes deformations. Fig(2) shows deformed shape with all displacements θ_A, θ_B and Δ. Final moments at end A and B are M_{AB} and M_{BA}.

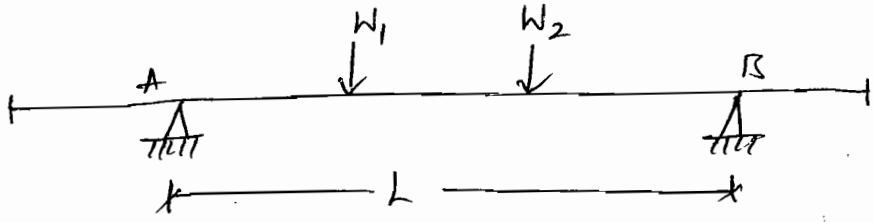


Fig 1 - Original shape of beam.

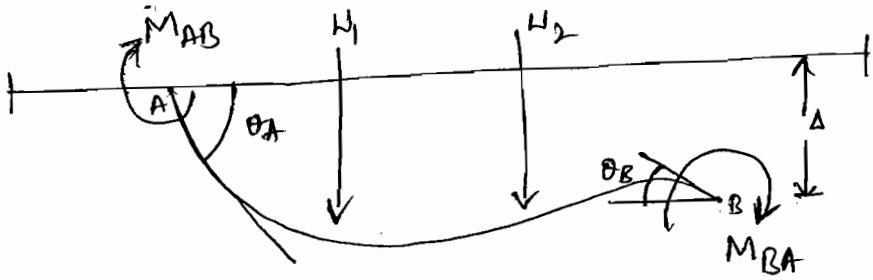
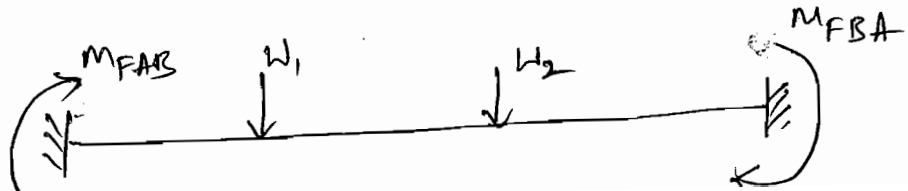


Fig2 - Deformed shape of beam.

The development of final moments and deformations are taken as follows.

1) Due to given loadings end moments M_{FAB} & M_{FBA} develop without any rotations at ends. These moments are similar to the end moments in a fixed beam & hence are called as fixed end moments. In fig 3, these are shown as positive



2) Settlement Δ takes place [illegible]

This is similar to the settlement of supports in fixed beams. From analysis of fixed beams we know, the end moments developed are $\frac{6EI\Delta}{l^2}$ as shown in figure 4.

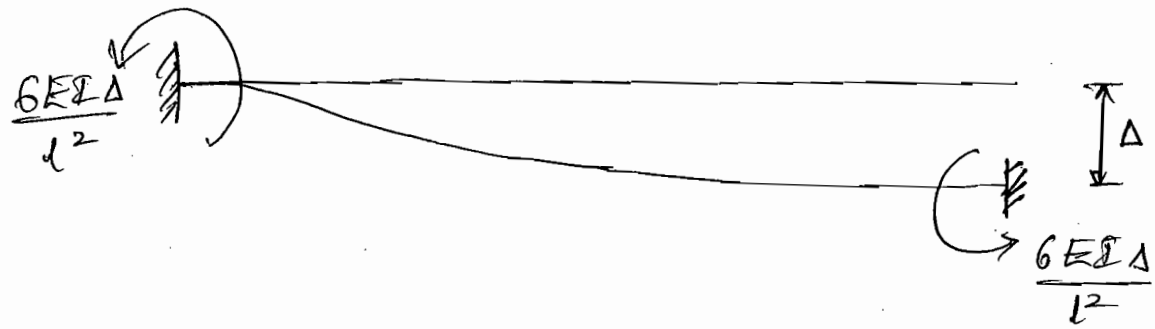


Fig 4 :- End moments due to Settlement.

3) Moment M'_{AB} comes into play in simply supported beam as shown in fig 5, to cause end rotations θ_{A1} & θ_{B1} at ~~end~~ A & B respectively

4) Moments M'_{BA} comes into play in simply supported beam shown in fig 6, the end rotations developed are θ_{A2} and θ_{B2}.

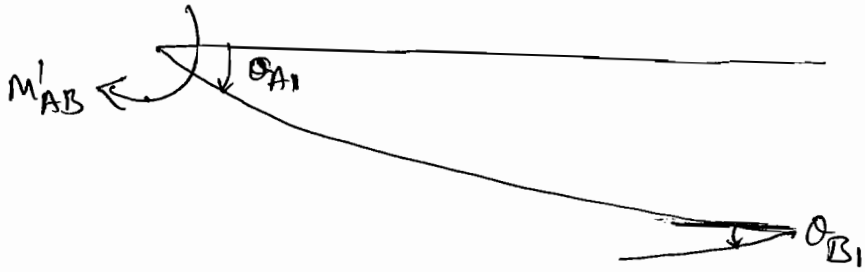


Fig 5 - End rotations due to M'_{AB}

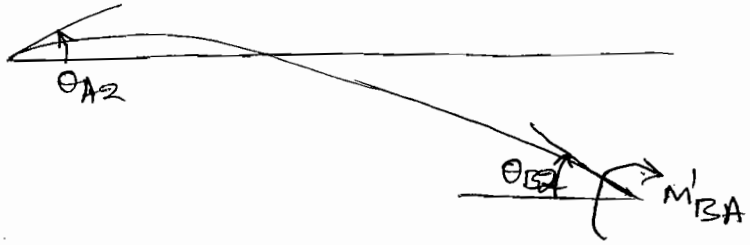


Fig 6. — End rotations due to M'_{BA}.

Moment M'_{AB} & M'_{BA} give final rotations θ_A & θ_B to the beam AB. To find the rotations due to applied moment 'M' in a beam without end rotation (Fig 7), Conjugate beam method may be used (Fig 8).

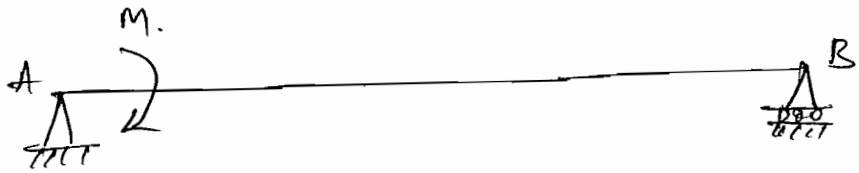


Fig 7. — SSB subjected to end moments.

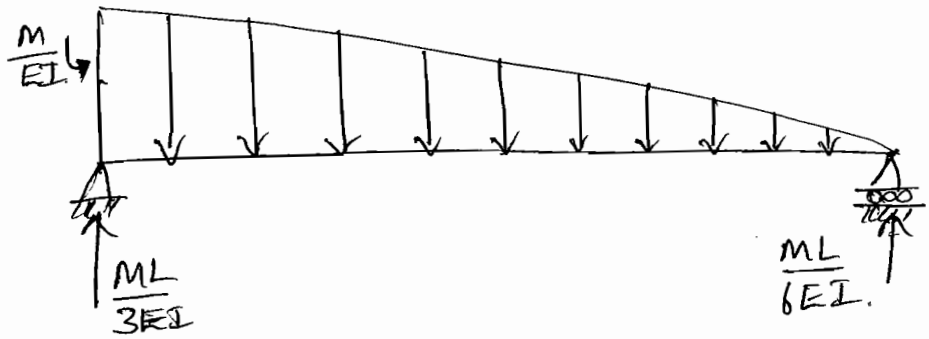


Fig 8. — Conjugate Beam

Hence refering fig (5) (6) & (8),

$$\theta_{A1} = \frac{M'_{AB} L}{3EI} \qquad \theta_{B1} = \frac{M'_{AB} L}{6EI}$$

$$\theta_{A2} = \frac{M'_{BA} L}{6EI} \qquad \theta_{B2} = \frac{M'_{BA} L}{3EI}$$

From fig 2, 5, & 6, it can be seen that.

$$\theta_A = \theta_{A1} - \theta_{A2} = \left(\frac{M'_{AB} \cdot L}{3EI}\right) - \left(\frac{M'_{BA}\, L}{6EI}\right)$$

$$\theta_B = \theta_{B2} - \theta_{B1} = \left(\frac{M'_{BA}\, L}{3EI}\right) - \left(\frac{M'_{AB} \cdot L}{6EI}\right)$$

$$\therefore \quad 2\theta_A + \theta_B = \frac{2M'_{AB} \cdot L}{3EI} - \cancel{\frac{2M'_{BA}\, L}{6EI}} + \cancel{\frac{M'_{BA} \cdot L}{3EI}} - \frac{M'_{AB} \cdot L}{6EI}$$

$$= \frac{M'_{AB}\, L}{EI}\left(\frac{2}{3} - \frac{1}{6}\right)$$

$$2\theta_A + \theta_B = \frac{1}{2} \cdot \frac{M'_{AB}\, L}{EI}$$

$$\boxed{M'_{AB} = \frac{2EI}{L}\left(2\theta_A + \theta_B\right)}$$

Illy $$\boxed{M'_{BA} = \frac{2EI}{L}\left(2\theta_B + \theta_A\right)}$$

Final moments shown in fig(2) are the sum of the moments shown in four stages, shown from fig 3 to 6.

$$\therefore \quad M_{AB} = M_{FAB} - \frac{6EI\Delta}{L^2} + M'_{AB}$$

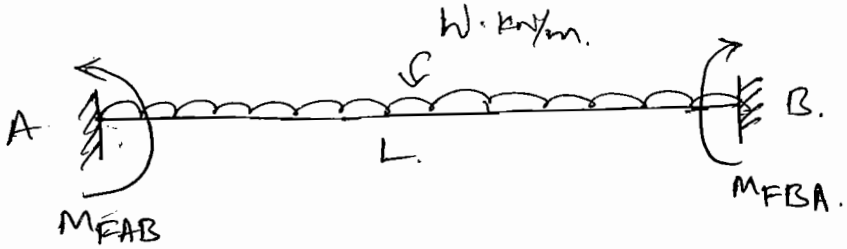
$$= M_{FAB} - \frac{6EI\Delta}{L^2} + \frac{2EI}{L}\left(2\theta_A + \theta_B\right)$$

$$\boxed{M_{AB} = M_{FAB} + \frac{2EI}{L}\left(2\theta_A + \theta_B - \frac{3\Delta}{L}\right)} \longrightarrow (1).$$

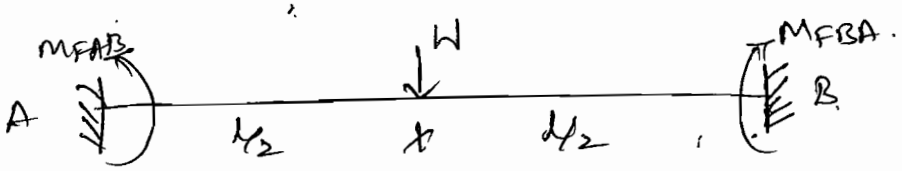
$$M_{BA} = M_{FBA} - \frac{6EI\Delta}{L^2} + M'_{BA}.$$

$$\boxed{M_{BA} = M_{FBA} + \frac{2EI}{L}\left(2\theta_B + \theta_A - \frac{3\Delta}{L}\right)} \longrightarrow (2).$$

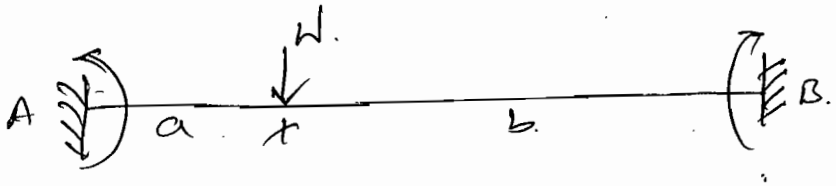
Fixed End Moments. (FEM).



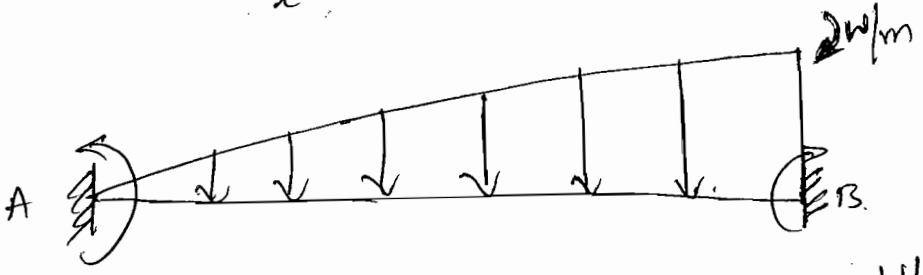
$$M_{FAB} = -\frac{Wl^2}{12}. \qquad M_{FBA} = +\frac{Wl^2}{12}$$

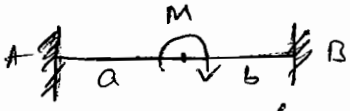


$$M_{FAB} = \frac{-WL}{8}. \qquad M_{FBA} = +\frac{WL}{8}.$$



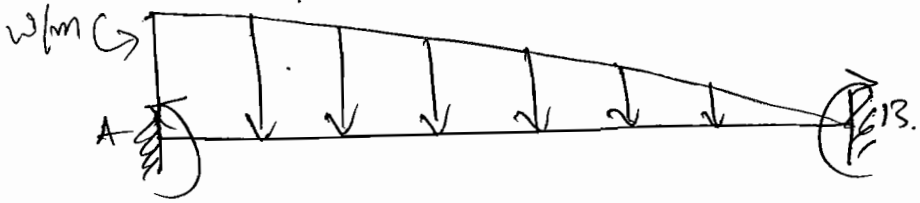
$$M_{FAB} = -\frac{Wab^2}{l^2} \qquad M_{FBA} = +\frac{Wa^2b}{l^2}$$





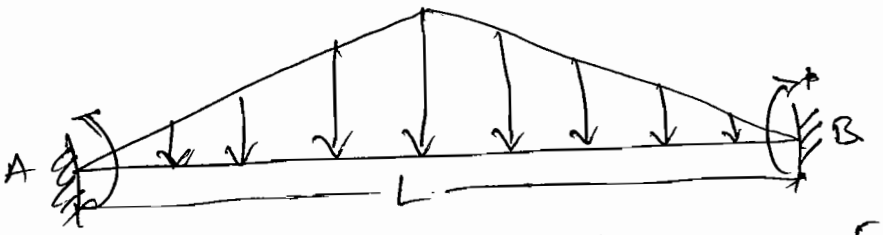
$$M_{FAB} = \frac{Mb(3a-l)}{l^2}$$

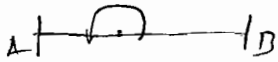
$$M_{FAB} = -\frac{Wl^2}{30} \qquad M_{FBA} = \frac{Wl^2}{20}. \qquad M_{FBA} = \frac{Ma(3b-l)}{l^2}$$



$$M_{FAB} = -\frac{Wl^2}{20} \qquad M_{FBA} = \frac{Wl^2}{30}.$$

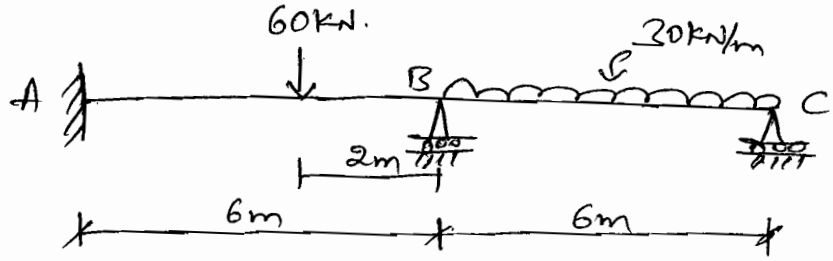
$$M_{FAB} = M_{FBA} = \frac{+M}{4}.$$





$$M_{FAB} = -\frac{5}{96}Wl^2 \qquad M_{FBA} = \frac{5}{96}Wl^2. \qquad M_{FAB} = M_{FBA} = -\frac{M}{4}$$

1) Analyse the CB shown in figure, draw BMD & elastic Curve



Sol:-

① FEM.

$$M_{FAB} = \frac{-Wab^2}{l^2} = \frac{-60\times4\times2^2}{6^2} = -26.67\,kN\text{-}m$$

$$M_{FBA} = \frac{+Wa^2b}{l^2} = \frac{+60\times4^2\times2}{6^2} = 53.33\,kN\text{-}m$$

$$M_{FBC} = \frac{-Wl^2}{12} = -\frac{30\times6^2}{12} = -90\,kN\text{-}m$$

$$M_{FCB} = \frac{+Wl^2}{12} = \frac{30\times6^2}{12} = 90\,kN\text{-}m$$

② S-D Eqn's

$$M_{AB} = M_{FAB} + \frac{2EI}{L}\left(2\cancel{\theta_A} + \theta_B - \cancel{\frac{3\Delta}{L}}\right)$$

$\theta_A = 0$ (fixed end) $\Delta = 0$.

$$M_{AB} = -26.67 + \frac{2EI}{6}(\theta_B)$$

$$= -26.67 + \frac{EI\theta_B}{3} \longrightarrow ①$$

$$M_{BA} = M_{FBA} + \frac{2EI}{L}\left(2\theta_B + \cancel{\theta_A} - \cancel{\frac{3\Delta}{L}}\right)$$

$$= 53.33 + \frac{2EI}{6}(2\theta_B)$$

$$= 53.33 + \frac{2}{3}EI\theta_B \longrightarrow ②$$

$$M_{BC} = M_{FBC} + \frac{2EI}{L}\left(2\theta_B + \theta_C - \cancel{\frac{3\Delta}{L}}\right)$$

$$= -90 + \frac{2EI}{6}(2\theta_B + \theta_C)$$

$$= -90 + \frac{2}{3}EI\theta_B + \frac{1}{3}EI\theta_C \longrightarrow ③$$

$$M_{CB} = M_{FCB} + \frac{2EI}{L}\left(2\theta_C + \theta_B - \frac{3\psi}{L}\right)$$

$$= 90 + \frac{2EI}{6}\left(2\theta_C + \theta_B\right)$$

$$= 90 + \frac{2}{3}EI\theta_C + \frac{1}{3}EI\theta_B \longrightarrow (4)$$

(3) Equilibrium Equ's.

Joint B $\rightarrow$ $\Sigma M_B = 0$

$$M_{BA} + M_{BC} = 0.$$

$$53.33 + \frac{2EI\theta_B}{3} - 90 + \frac{2}{3}EI\theta_B + \frac{1}{3}EI\theta_C = 0.$$

$$1.32EI\theta_B + 0.33EI\theta_C = 36.67 \longrightarrow (5)$$

Joint C $\rightarrow$ $\Sigma M_C = 0.$

$$M_{CB} = 0.$$

$$0.33EI\theta_B + 0.66EI\theta_C = -90 \longrightarrow (6)$$

Solving equ's (5) & (6)

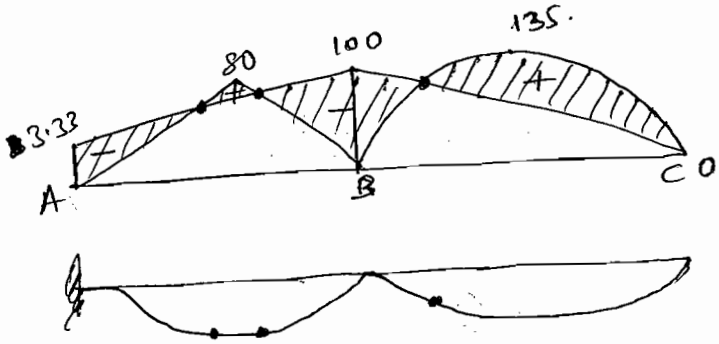
$EI\theta_B = 70$ $\qquad$ $EI\theta_C = -170$

(4) End Moments.

Subs $EI\theta_B$ & $EI\theta_C$ in equ's (1), (2), (3) & (4)

$M_{AB} = -38.33$ kN-m $\qquad$ $M_{BC} = -100$ kN-m

$M_{BA} = 100$ kN-m $\qquad$ $M_{CB} = 0$



BMD.

Span AB.

$$\frac{Wab}{l} = \frac{60\times4\times2}{6}$$

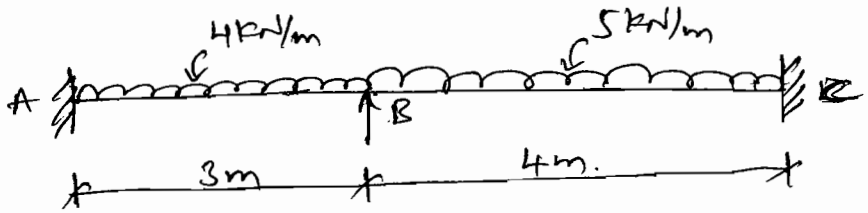
$$= 80 \text{ kN-m}$$

Span BC.

$$\frac{Wl^2}{8} = \frac{30\times6^2}{8}.$$

$$= 135 \text{ kN-m}$$

2) Analyse the CB [illegible] curve



Sol:-

① FEM.

$$M_{AB} = \frac{-wl^2}{12} = -\frac{4\times 3^2}{12} = -3 \text{ kN-m}$$

$$M_{BA} = \frac{4\times 3^2}{12} = 3 \text{ kN-m}$$

$$M_{BC} = \frac{-wl^2}{12} = \frac{-5\times 4^2}{12} = -6.67 \text{ kN-m}$$

$$M_{CB} = \frac{5\times 4^2}{12} = 6.67 \text{ kN-m}$$

② S-D Equ's.

$$M_{AB} = M_{FAB} + \frac{2EI}{L}\left(2\theta_A + \theta_B - \frac{3\Delta}{L}\right)$$

$$\theta_A = 0, \quad \Delta = 0.$$

$$M_{AB} = -3 + \frac{2EI}{3}(\theta_B)$$

$$M_{AB} = -3 + 0.66EI\theta_B \longrightarrow ①$$

$$M_{BA} = 3 + \frac{2EI}{3}\left(2\theta_B + \cancel{\theta_A} - \cancel{\frac{3\Delta}{3}}\right)$$

$$= 3 + 1.33EI\theta_B \longrightarrow ②$$

$$M_{BC} = -6.67 + \frac{2EI}{4}\left(2\theta_B + \cancel{\theta_C} - \cancel{\frac{3\Delta}{L}}\right)$$

$$\theta_C = 0.$$

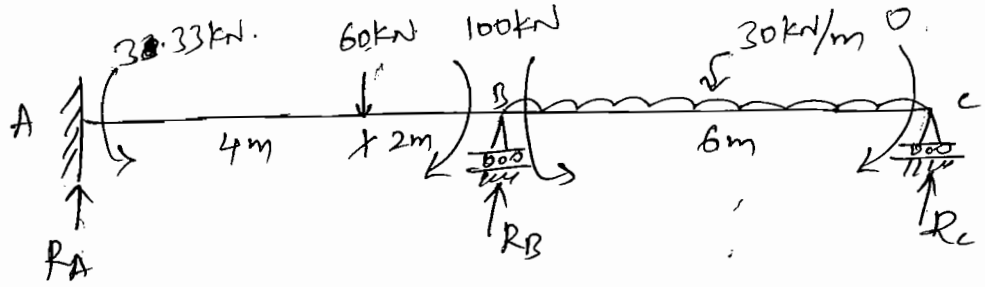
$$= -6.67 + EI\theta_B \longrightarrow ③$$

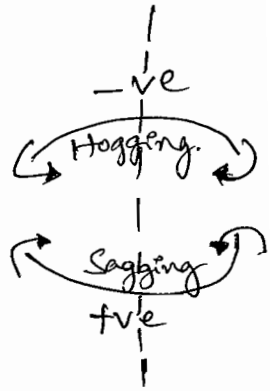
$$M_{CB} = 6.67 + \frac{2EI}{4}\left(\cancel{2\theta_C} + \theta_B - \cancel{\frac{3\Delta}{L}}\right)$$

$$= 6.67 + 0.5EI\theta_B \longrightarrow ④$$

SFD for problems ① & ②

①





Reactions.

$\Sigma M_B = 0$ (LHS)

$(R_A \times 6) - \cancel{33.33}\ 3.33 - (60 \times 2) + 100 = 0.$

$\boxed{R_A = 3.88\ kN}$

$\Sigma M_B = 0$ (RHS)

$(R_C \times 6) - 0 - (30 \times 6 \times 6/2) + 100 = 0.$

$\boxed{R_C = 73.33\ kN}$

$\Sigma M_A = 0$ (RHS)

$(R_C \times 12) - 0 - (30 \times 6 \times 9) + 100 - 100 + (R_B \times 6) - (60 \times 4) + \cancel{33.33}\ 3.33 = 0.$

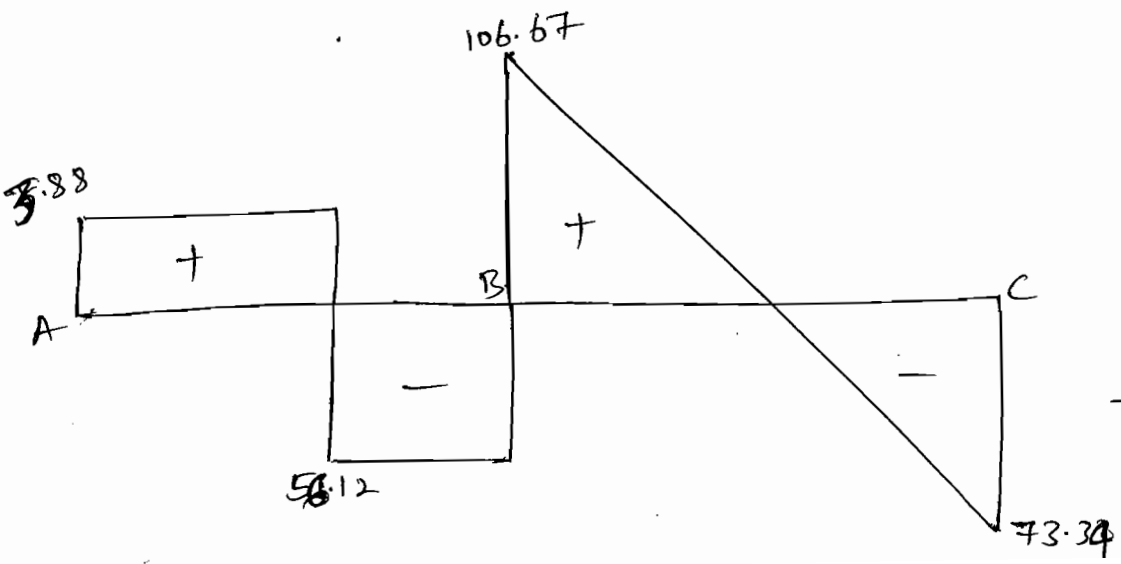
$\boxed{R_B = 162.78\ kN}$

(or)

$\Sigma V = 0$

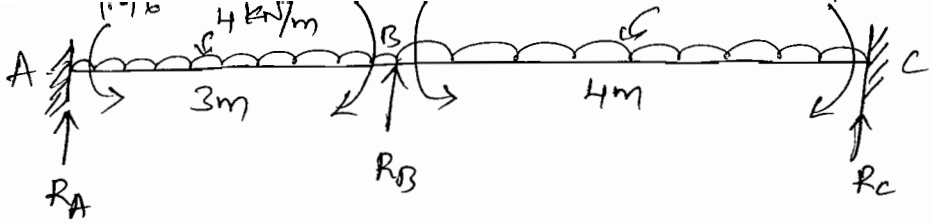
$R_A + R_B + R_C = 60 + (30 \times 6) = 240\ kN.$

$\boxed{R_B = \cancel{157.79}\ 162.78\ kN}$



SFD.

(2)



Reactions

$\Sigma M_B = 0$ (LHS)

$(R_A \times 3) - 1.96 - (4 \times 3 \times \frac{3}{2}) + 5.1 = 0.$

$\boxed{R_A = 4.95 \text{ kN.}}$

$\Sigma M_B = 0$ (RHS)

$(R_C \times 4) - 7.457 - (5 \times 4 \times \frac{4}{2}) + 5.1 = 0.$

$\boxed{R_C = 10.58 \text{ kN}}$

$\Sigma M_A = 0$ (RHS)

$(R_C \times 7) - 7.457 - (5 \times 4 \times 5) + (R_B \times 3) + 5.1 - 5.1 - (4 \times 3 \times \frac{3}{2}) + 1.96 = 0$

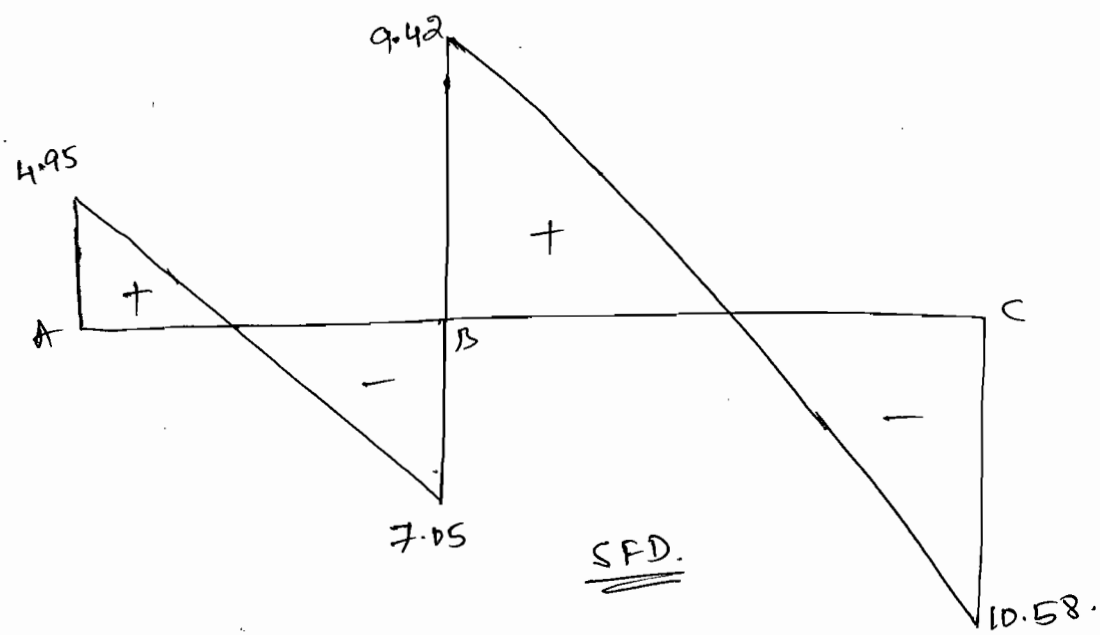
$\boxed{R_B = 16.47 \text{ kN}}$

(or)

$\Sigma V = 0.$

$R_A + R_B + R_C = (4 \times 3) + (5 \times 4) = 32.$

$\boxed{R_B = 16.47 \text{ kN.}}$



SFD.

③ Equil Equs.

At Joint B.

$\Sigma M_B = 0.$

$M_{BA} + M_{BC} = 0$

$3 + 1.33EI\theta_B - 6.67 + EI\theta_B = 0.$

$2.33EI\theta_B = 3.67$

$$\boxed{EI\theta_B = 1.575}$$

④ End Moments.

Subs $EI\theta_B$ in equs ①, ②, ③ & ④

$M_{AB} = -3 + 0.66(1.575) = -1.96$ kN-m

$M_{BA} = 3 + 1.33(1.575) = 5.10$ kN-m

$M_{BC} = -6.67 + 1.575 = -5.10$ kN-m

$M_{CB} = 6.67 + 0.5(1.575) = 7.457$ kN-m

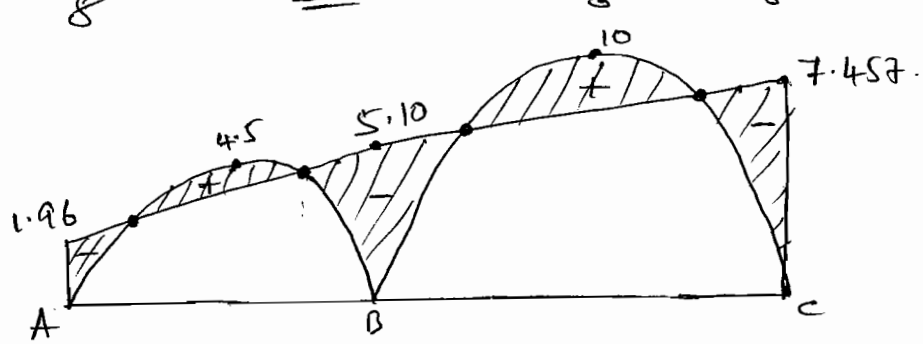
BMD.

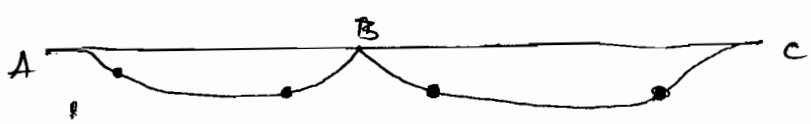
Span AB

$= \frac{Wl^2}{8} = \frac{4 \times 3^2}{8} = 4.5$ kN-m

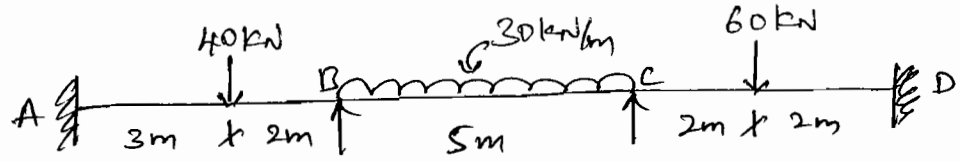
Span BC

$\frac{Wl^2}{8} = \frac{5 \times 4^2}{8} = 10$ kN-m





5/ Analyse



Sol:

① FEM.

$$M_{FAB} = -\frac{Wab^2}{l^2} = -\frac{40\times3\times2^2}{(5)^2} = -19.2\,kN\text{-}m$$

$$M_{FBA} = +\frac{Wa^2b}{l^2} = \frac{40\times3^2\times2}{5^2} = 28.8\,kN\text{-}m$$

$$M_{FBC} = -\frac{Wl^2}{12} = -\frac{30\times5^2}{12} = -62.5\,kN\text{-}m$$

$$M_{FCB} = +\frac{Wl^2}{12} = 62.5\,kN\text{-}m$$

$$M_{FCD} = -\frac{Wl}{8} = -\frac{60\times4}{8} = -30\,kN\text{-}m$$

$$M_{FDC} = +\frac{Wl}{8} = 30\,kN\text{-}m$$

② S-D Equ's.

$$M_{AB} = M_{FAB} + \frac{2EI}{L}\left(2\theta_A + \theta_B - \frac{3\Delta}{L}\right)$$

$$\theta_A = 0,\ \theta_D = 0.,\ \Delta = 0.$$

$$M_{AB} = -19.2 + \frac{2EI}{5}\left(2\cancel{\theta_A}^{0} + \theta_B - \cancel{\frac{3\Delta}{5}}^{0}\right)$$

$$= -19.2 + 0.4EI\theta_B \quad \text{——} ①$$

$$M_{BA} = 28.8 + \frac{2EI}{5}(2\theta_B)$$

$$= 28.8 + 0.8EI\theta_B \quad \text{——} ②$$

$$M_{BC} = -62.5 + \frac{2EI}{5}(2\theta_B + \theta_C)$$

$$= -62.5 + 0.8EI\theta_B + 0.4EI\theta_C \quad \text{——} ③$$

$$M_{CB} = 62.5 + 2EI/5\,(2\theta_C + \theta_B)$$

$$= 62.5 + 0.8EI\theta_C + 0.4EI\theta_B \quad \text{——} ④$$

$M_{CD} = -30 + \frac{2EI}{4}(\theta_c)$

$= -30 + EI\theta_c$ —— (5)

$M_{DC} = 30 + \frac{2EI}{4}(\theta_c)$

$= 30 + 0.5EI\theta_c$ —— (6)

(3) Equm Equ's.

At Support B,

$M_{BA} + M_{BC} = 0.$

$28.8 + 0.8EI\theta_B - 62.5 + 0.8EI\theta_B + 0.4EI\theta_c = 0.$

$1.6EI\theta_B + 0.4EI\theta_c = 33.7$ —— (7)

At Support C,

$M_{CB} + M_{CD} = 0$

$62.5 + 0.4EI\theta_B + 0.8EI\theta_c - 30 + EI\theta_c = 0$

$0.4EI\theta_B + 1.8EI\theta_c = -32.5$ —— (8)

Solving equ's (7) & (8),

$EI\theta_B = 27.08.$ $EI\theta_c = -24.07.$

(4) End Moments.

Subs $EI\theta_B$ & $EI\theta_c$ in equ's (1) to (6).

$M_{AB} = -8.36$ kN-m

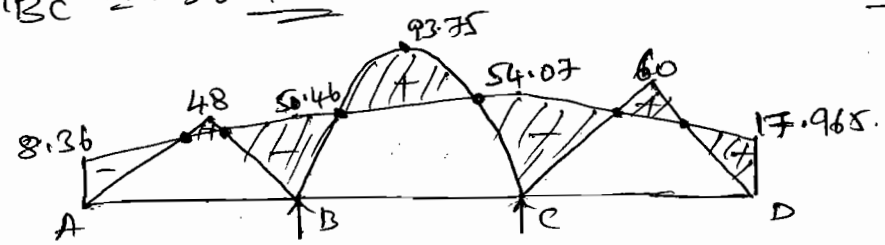
$M_{BA} = 50.46$ kN-m

$M_{BC} = -50.46$ kN-m

$M_{CB} = 54.07$ kN-m

$M_{CD} = -54.07$ kN-m

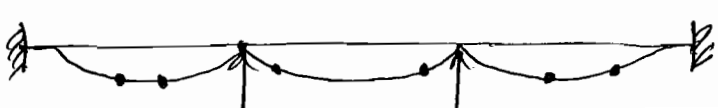
$M_{DC} = 17.965$ kN-m



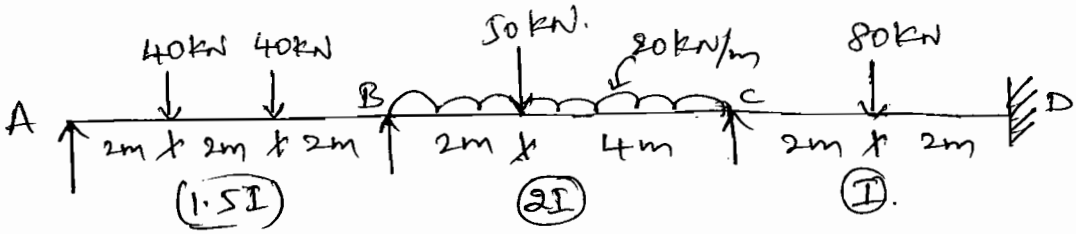
$\frac{Wab}{l} = 48$ kN-m

$\frac{Wl^2}{8} = 93.75$ kN-m

$\frac{Wl}{4} = 60$ kN-m



4. Analyse



Soln

① FEM.

$$M_{FAB} = -\frac{Wab^2}{l^2} = -\left[\frac{40\times2\times4^2}{(6)^2} + \frac{40\times4\times2^2}{6^2}\right] = -53.33\ kN\text{-}m$$

$$M_{FBA} = +\frac{Wa^2b}{l^2} = \left[\frac{40\times2^2\times4}{6^2} + \frac{40\times4^2\times2}{6^2}\right] = 53.33\ kN\text{-}m$$

$$M_{FBC} = -\frac{Wl^2}{12} - \frac{Wab^2}{l^2} = -\frac{50\times2^2\times4^2}{6^2} - \frac{20\times6^2}{12} = -104.44\ kN\text{-}m$$

$$M_{FCB} = +\frac{Wl^2}{12} + \frac{Wa^2b}{l^2} = \frac{20\times6^2}{12} + \frac{50\times2^2\times4}{(6)^2} = 82.22\ kN\text{-}m$$

$$M_{FCD} = -\frac{Wl}{8} = -\frac{80\times4}{8} = -40\ kN\text{-}m$$

$$M_{FDC} = \frac{Wl}{8} = \frac{80\times4}{8} = 40\ kN\text{-}m$$

② S.D. Eqns.

$\theta_D = 0, \quad \Delta = 0.$

$$M_{AB} = M_{FAB} + \frac{2EI}{L}\left(2\theta_A + \theta_B - \frac{3\Delta}{L}\right)$$

$$= -53.33 + \frac{2(1.5EI)}{6}(2\theta_A + \theta_B)$$

$$= -53.33 + EI\theta_A + 0.5EI\theta_B \quad \text{——— ①.}$$

$$M_{BA} = 53.33 + \frac{2(1.5EI)}{6}(2\theta_B + \theta_A)$$

$$= 53.33 + EI\theta_B + 0.5EI\theta_A \quad \text{——— ②.}$$

$$M_{BC} = -104.44 + \frac{2(2EI)}{6}(2\theta_B + \theta_C)$$

$$= -104.44 + 1.33EI\theta_B + 0.67EI\theta_C \quad \text{——— ③}$$

$$M_{CB} = 82.22 + \frac{2(2EI)}{6}(2\theta_C + \theta_B)$$

$M_{CD} = -40 + \frac{2EI}{4}(2\theta_C + \theta_D)$

$= -40 + EI\theta_C$ —— (5)

$M_{DC} = 40 + \frac{2EI}{4}(2\theta_D + \theta_C)$

$= 40 + 0.5EI\theta_C$ —— (6).

(3) Equ^n Condition.

At joint A,

$M_{AB} = 0.$

$EI\theta_A + 0.5EI\theta_B = 53.33$ —— (7)

At joint B,

$M_{BA} + M_{BC} = 0.$

$53.33 + 0.5EI\theta_A + EI\theta_B - 104.44 + 1.33EI\theta_B + 0.67EI\theta_C = 0$

$0.5EI\theta_A + 2.33EI\theta_B + 0.67EI\theta_C = 51.11$ —— (8)

At joint C,

$M_{CB} + M_{CD} = 0.$

$82.22 + 1.33EI\theta_C + 0.67EI\theta_B - 40 + EI\theta_C = 0.$

$0.67EI\theta_B + 2.33EI\theta_C = -42.22.$ —— (9)

Solving eqns (7), (8) and (9).

$EI\theta_A = 43.64$ $EI\theta_B = 19.38.$ $EI\theta_C = -23.69.$

(4) Final Moments.

Subs $EI\theta_A$, $EI\theta_B$ & $EI\theta_C$ in eqns (1) to (6).

$M_{AB} = 0.$

$M_{BA} = 94.53$ kN-m

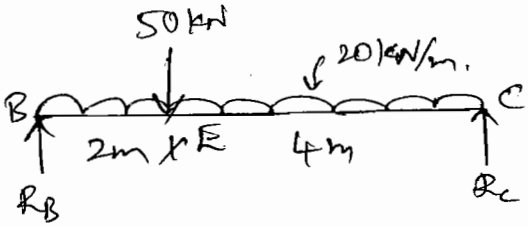
$M_{BC} = -94.53$ kN-m

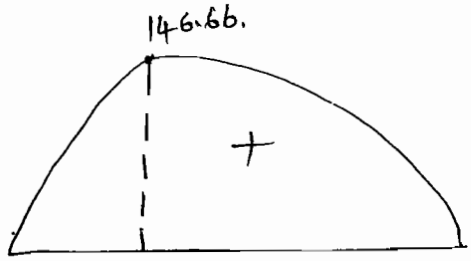
$M_{CB} = 63.69$ kN-m

$M_{CD} = -63.69$ kN-m

$M_{DC} = 28.15$ kN-m

BMD for Span BC





$\Sigma V = 0.$

$R_B + R_C = 50 + (20 \times 6)$

$R_B + R_C = 170.$

$\Sigma M_B = 0.$

$-R_C \times 6 + \frac{20 \times 6^2}{2} + 50 \times 2 = 0.$

$R_C = 76.67$ kN.

$R_B = 93.33$ kN.

$M_E = (93.33 \times 2) - \left(\frac{20 \times 2^2}{2}\right)$

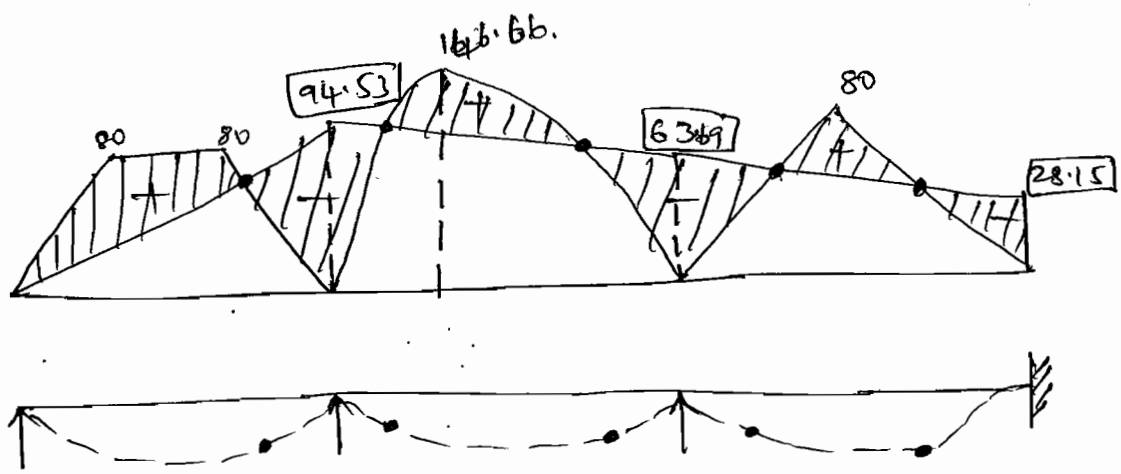
$= 146.66$ kN-m

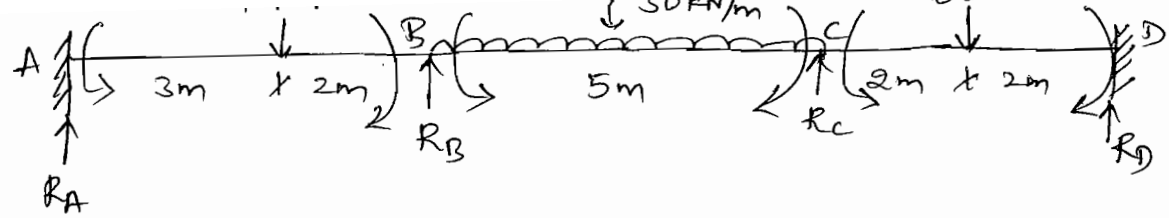
AB

$Wa = 40 \times 2 = 80$ kN-m

CD

$\frac{Wl}{4} = \frac{80 \times 4}{4} = 80$ kN-m





Reactions.

$\Sigma M_B = 0$ (LHS)

$(R_A \times 5) - 8.36 - (40 \times 2) + 50.46 = 0.$

$\boxed{R_A = 7.58 \text{ kN}}$

$\Sigma M_C = 0$ (RHS)

$(R_D \times 4) - 17.965 - (60 \times 2) + 54.07 = 0.$

$\boxed{R_D = 20.97 \text{ kN}}$

$\Sigma M_C = 0$ (LHS).

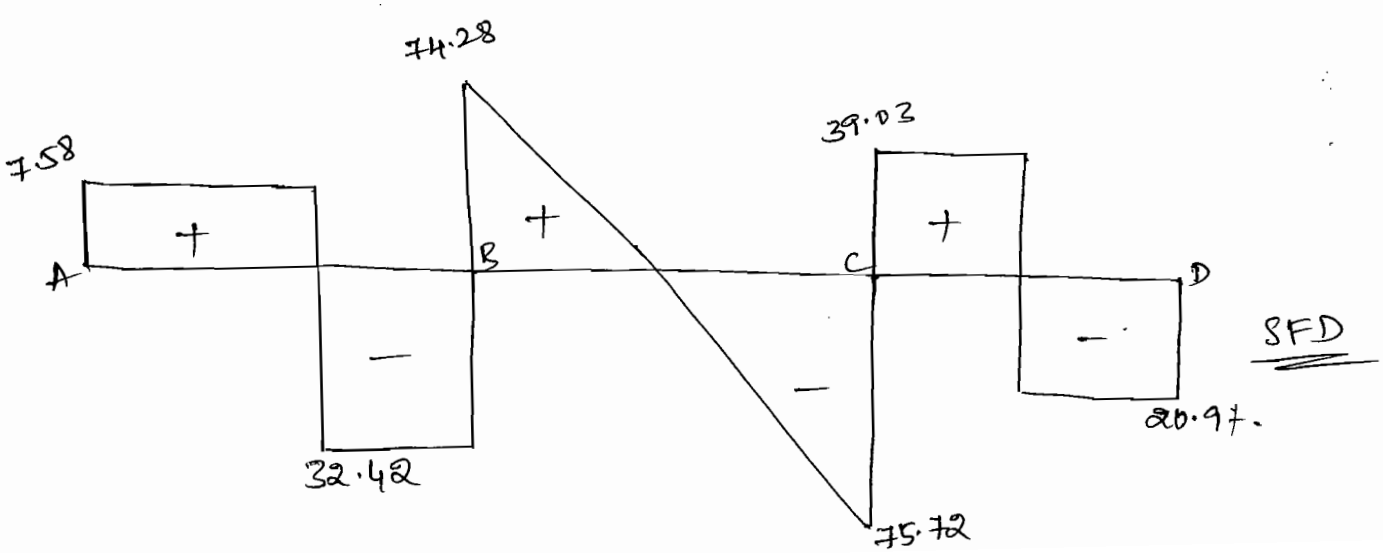
$(R_A \times 10) - 8.36 - (40 \times 7) - 50.46 + 50.46 - (30 \times 5 \times \frac{5}{2}) + 54.07 + R_B \times 5 = 0.$

$\boxed{R_B = 106.7 \text{ kN}}$

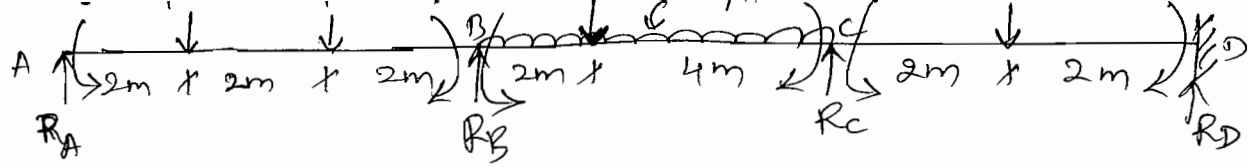
$\Sigma M_B = 0$ (RHS)

$(R_D \times 9) - 17.965 - (60 \times 7) + 54.07 - 54.07 - (30 \times 5 \times \frac{5}{2}) + (R_C \times 5) + 50.46 = 0.$

$\boxed{R_C = 114.75 \text{ kN}}$



SFD



Reactions

$\Sigma V = 0.$

$R_A + R_B + R_C + R_D = 40 + 40 + 50 + (20 \times 6) + 80 = 330. \rightarrow (1)$

$\Sigma M_B = 0$ (LHS)

$R_A \times 6 - 0 - (40 \times 4) - (40 \times 2) + 94.53 = 0.$

$\boxed{R_A = 24.245 \text{ kN}}$

$\Sigma M_C = 0$ (LHS)

$(24.245 \times 12) - 0 - (40 \times 10) - (40 \times 8) - 94.53 + 94.53 + (R_B \times 6)$

$-(50 \times 4) - (20 \times 6 \times \frac{6}{2}) + 63.69 = 0$

$\boxed{R_B = 154.22 \text{ kN}}$

$\Sigma M_C = 0$ (RHS)

$(R_D \times 4) - 28.15 - (80 \times 2) + 63.69 = 0$

$\boxed{R_D = 31.115 \text{ kN}}$

(1) $\Rightarrow$

$R_C = 330 - (24.245 + 154.22 + 31.115)$

$\boxed{R_C = 120.42 \text{ kN}}$

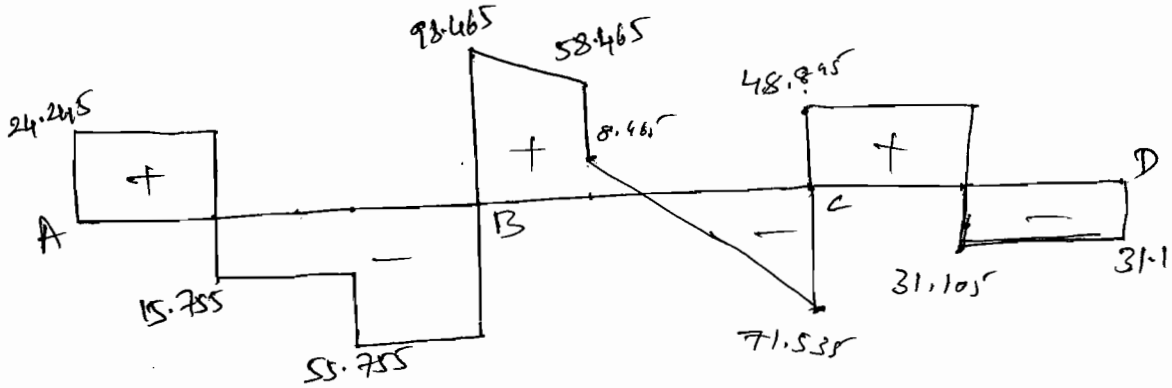
(or) $\Sigma M_D = 0.$ (LHS)

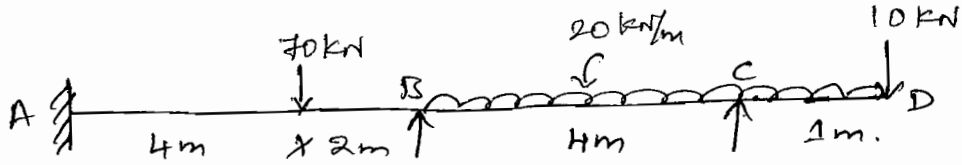
$(24.245 \times 16) - 0 - (40 \times 14) - (40 \times 12) - 94.53 + 94.53 + (154.22 \times 10)$

$-(50 \times 8) - (20 \times 6 \times 7) - 63.69 + 63.69 + (R_C \times 4) - (80 \times 2)$

$+ 28.15 = 0.$

$\boxed{R_C = 120.43 \text{ kN}}$





Sol:

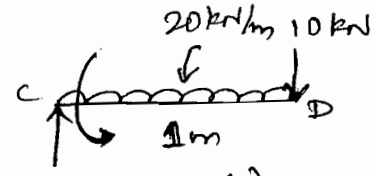
① FEM.

$$M_{FAB} = -\frac{Wab^2}{l^2} = -\frac{70\times4\times2^2}{6^2} = -31.11\ kN\text{-}m$$

$$M_{FBA} = \frac{Wa^2b}{l^2} = \frac{70\times4^2\times2}{6^2} = 62.22\ kN\text{-}m$$

$$M_{FBC} = -\frac{Wl^2}{12} = -\frac{20\times4^2}{12} = -26.67\ kN\text{-}m$$

$$M_{FCB} = \frac{Wl^2}{12} = 26.67\ kN\text{-}m$$



$$M_{CD} = -(10\times1 + 20\times1\times\tfrac{1}{2})$$

$$= -20\ kN\text{-}m.$$ (-ve sign for resisting moment)

② S.D Equ's.

$\theta_A = 0, \quad \Delta = 0$

$$M_{AB} = -31.11 + \frac{2EI}{6}(\theta_B) = -31.11 + 0.33EI\theta_B \quad \text{—— ①}$$

$$M_{BA} = 62.22 + \frac{2EI}{6}(2\theta_B) = 62.22 + 0.66EI\theta_B \quad \text{—— ②}$$

$$M_{BC} = -26.67 + \frac{2EI}{4}(2\theta_B + \theta_C) = -26.67 + EI\theta_B + 0.5EI\theta_C \quad \text{—— ③}$$

$$M_{CB} = 26.67 + \frac{2EI}{4}(2\theta_C + \theta_B) = 26.67 + 0.5EI\theta_B + EI\theta_C \quad \text{—— ④}$$

There is No S.D equ for Overhanging portion CD.

③ Equm Condition.

At joint B, $M_{BA} + M_{BC} = 0$

$$62.22 + 0.66EI\theta_B - 26.67 + EI\theta_B + 0.5EI\theta_C = 0.$$

$$1.66EI\theta_B + 0.5EI\theta_C = -35.55 \quad \text{—— ⑤}$$

At joint C, $M_{CB} + M_{CD} = 0.$

solving, equs ⑤ & ⑥

$$EI\theta_B = -22.84 \qquad EI\theta_C = 4.75$$

④ Final Moments.

Subs, $EI\theta_B$ & $EI\theta_C$ in equs ① to ④

$$M_{AB} = 0.33(-22.84) - 31.11 = -38.64 \text{ kN-m}$$

$$M_{BA} = 0.66(-22.84) + 62.22 = 47.14 \text{ kN-m}$$

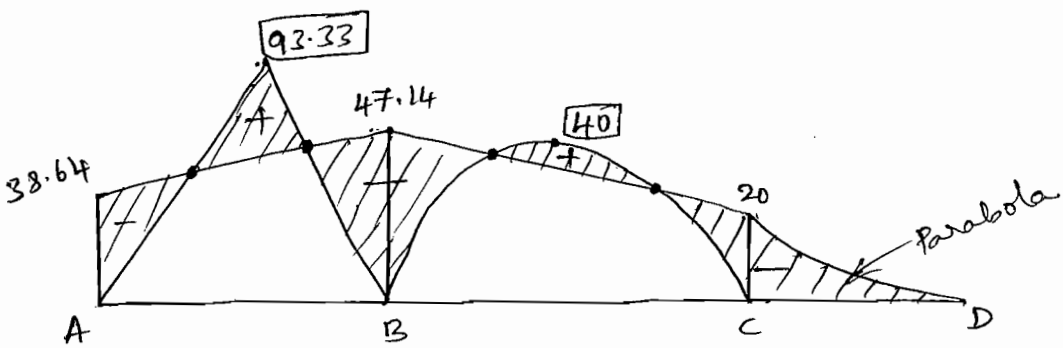
$$M_{BC} = (-22.84) + 0.5(4.75) - 26.67 = -47.14 \text{ kN-m}$$

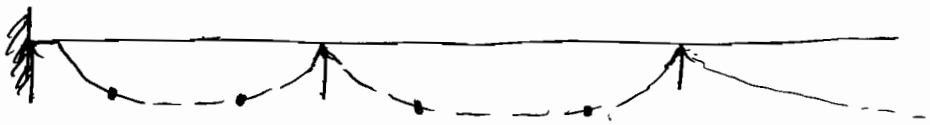
$$M_{CB} = 4.75 + 0.5(-22.84) + 26.67 = 20 \text{ kN-m}$$

$$M_{CD} = -20 \text{ kN-m}.$$

BMD.

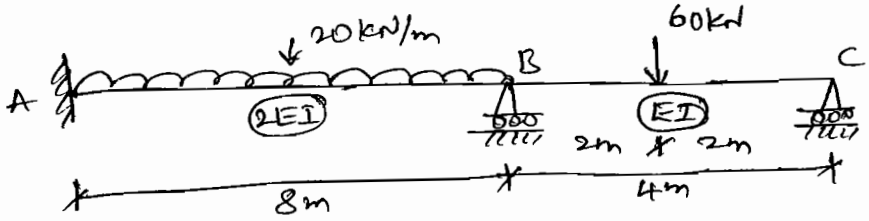
$$\frac{Wab}{l} = \frac{70 \times 4 \times 2}{6} = 93.33 \text{ kN-m} \qquad \frac{wl^2}{8} = \frac{20 \times 4^2}{8} = 40 \text{ kN-m}$$



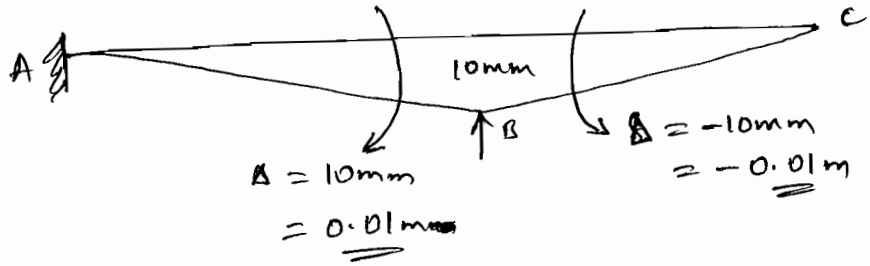


EC

(6) Analyse the [illegible] by 10mm. Given $EI = 4000\ kNm^2$. Draw BMD & EC



Soln



① FEM.

$$M_{FAB} = -\frac{20\times 8^2}{12} = -106.67\ kN\text{-}m$$

$$M_{FBA} = \frac{20\times 8^2}{12} = 106.67\ kN\text{-}m$$

$$M_{FBC} = -\frac{60\times 4}{8} = -30\ kN\text{-}m$$

$$M_{FCB} = \frac{60\times 4}{8} = 30\ kN\text{-}m$$

② S-D Eqn's, $\theta_A = 0$, $\Delta = +0.01m$.

$$M_{AB} = -106.67 + \frac{2(4000\times 2)}{8}\left(\theta_B - \frac{3(0.01)}{8}\right)$$

$$= -106.67 + 2000\theta_B - 7.5$$

$$= 2000\theta_B - 114.17 \quad \text{——— (1)}$$

$$M_{BA} = 106.67 + \frac{2(2\times 4000)}{8}\left(2\theta_B - \frac{3(0.01)}{8}\right)$$

$$= 106.67 + 4000\theta_B - 7.5$$

$$= 4000\theta_B + 99.17 \quad \text{——— (2)}$$

$\Delta = -0.01$

$$M_{BC} = -30 + \frac{2(4000)}{4}\left(2\theta_B + \theta_C - \frac{3(-0.01)}{4}\right)$$

$$= -30 + 4000\theta_B + 2000\theta_C + 15$$

$$= 4000\theta_B + 2000\theta_C - 15 \quad \text{——— (3)}$$

$$= 30 + 2000\,\theta_B + 4000\,\theta_C + 15.$$

$$= 2000\,\theta_B + 4000\,\theta_C + 45 \quad \text{——— (4).}$$

③ Equ^m Conditions.

At joint B, $M_{BA} + M_{BC} = 0.$

$$4000\,\theta_B + 99.17 + 4000\,\theta_B + 2000\,\theta_C - 15 = 0.$$

$$8000\,\theta_B + 2000\,\theta_C = -84.17 \quad \text{——— (5)}$$

At joint C, $M_{CB} = 0.$

$$2000\,\theta_B + 4000\,\theta_C = -45 \quad \text{——— (6)}$$

Solving equ's (5) & (6)

$$\boxed{\theta_B = -8.81\times10^{-3}\ \text{Radians}} \quad \boxed{\theta_C = -6.845\times10^{-3}\ \text{Radians}}$$

④ Final Moments.

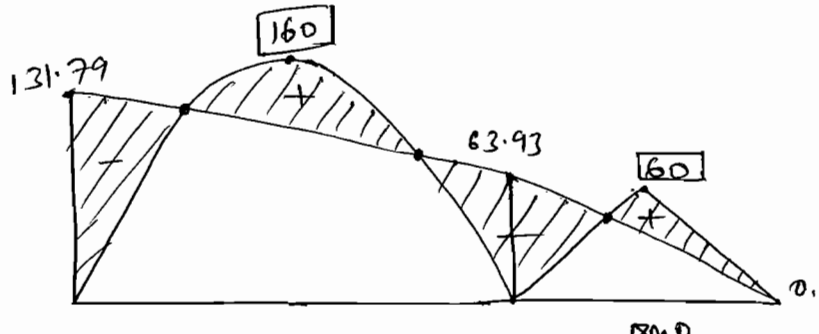
Subs θ_B & θ_C in equ's (1) to (4).

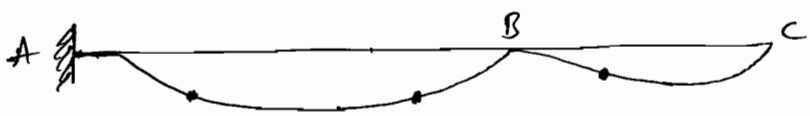
$M_{AB} = -131.79$ kN-m

$M_{BA} = 63.93$ kN-m

$M_{BC} = -63.93$ kN-m

$M_{CB} = 0.$





BM.

AB

$$\frac{Wl^2}{8} = \frac{20\times8^2}{8}$$

$$= 160\ \text{kN-m}$$

BC

$$\frac{Wl}{4} = \frac{60\times4}{4}$$

$$= 60\ \text{kN-m}$$

EC

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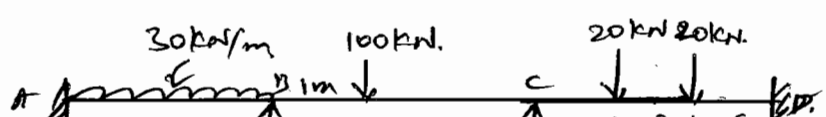
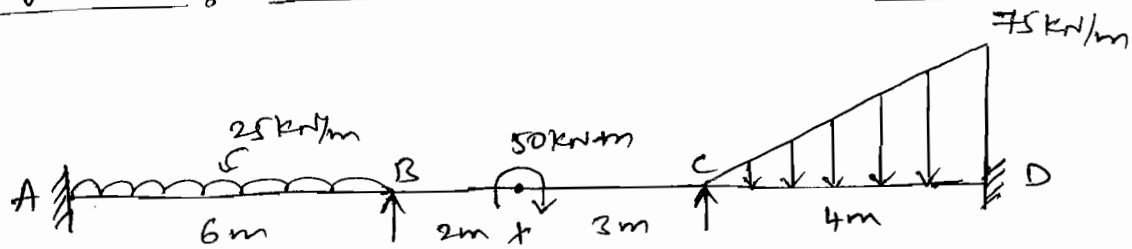
②.



Support A rotates through (1/200) radians

Support B settles by 20mm

(4) Analyse the beam shown and draw BMD & SFD.



Soln (1) FEM.

$$M_{FAB} = -\frac{Wl^2}{12} = \frac{-25\times 6^2}{12} = -75\ kN\text{-}m$$

$$M_{FBA} = 75\ kN\text{-}m$$

$$M_{FBC} = +\frac{Mb(3a-l)}{l^2} = \frac{50\times 3[3(2)-5]}{5^2} = 6\ kN\text{-}m$$

$$M_{FCB} = \frac{Ma(3b-l)}{l^2} = \frac{50\times 2[3(3)-5]}{5^2} = 16\ kN\text{-}m$$

$$M_{FCD} = -\frac{Wl^2}{30} = \frac{-75\times 4^2}{30} = -40\ kN\text{-}m$$

$$M_{FDC} = +\frac{Wl^2}{20} = \frac{75\times 4^2}{20} = 60\ kN\text{-}m$$

(2) S-D Equ's.

$$\theta_A = \theta_D = 0. \qquad \Delta = 0.$$

$$M_{AB} = -75 + \frac{2EI}{6}\cdot(\theta_B) = 0.33EI\theta_B - 75 \quad \text{—— (1)}$$

$$M_{BA} = 75 + \frac{2EI}{6}(2\theta_B) = 0.66EI\theta_B + 75 \quad \text{—— (2)}$$

$$M_{BC} = 6 + \frac{2EI}{5}(2\theta_B + \theta_C) = 0.8EI\theta_B + 0.4EI\theta_C + 6 \quad \text{—— (3)}$$

$$M_{CB} = 16 + \frac{2EI}{5}(2\theta_C + \theta_B) = 0.4EI\theta_B + 0.8EI\theta_C + 16 \quad \text{—— (4)}$$

$$M_{CD} = -40 + \frac{2EI}{4}(2\theta_C) = EI\theta_C - 40 \quad \text{—— (5)}$$

$$M_{DC} = 60 + \frac{2EI}{4}(\theta_C) = 0.5EI\theta_C + 60 \quad \text{—— (6)}$$

(3) Equm Conditions.

At Joint B, $M_{BA} + M_{BC} = 0$

$$0.66EI\theta_B + 75 + 0.8EI\theta_B + 0.4EI\theta_C + 6 = 0.$$

$$0.4EI\theta_B + 0.8EI\theta_C + 16 + EI\theta_C - 40 = 0.$$

$$0.4EI\theta_B + 1.8EI\theta_C = 24 \quad \text{——— (8)}$$

Solving equ's (7) & (8)

$$EI\theta_B = -62.96. \qquad EI\theta_C = 27.32.$$

(4) Final Moments.

Subs the values of $EI\theta_B$ & $EI\theta_C$ in equ's (1) to (6).

$M_{AB} = -95.77$ kN-m

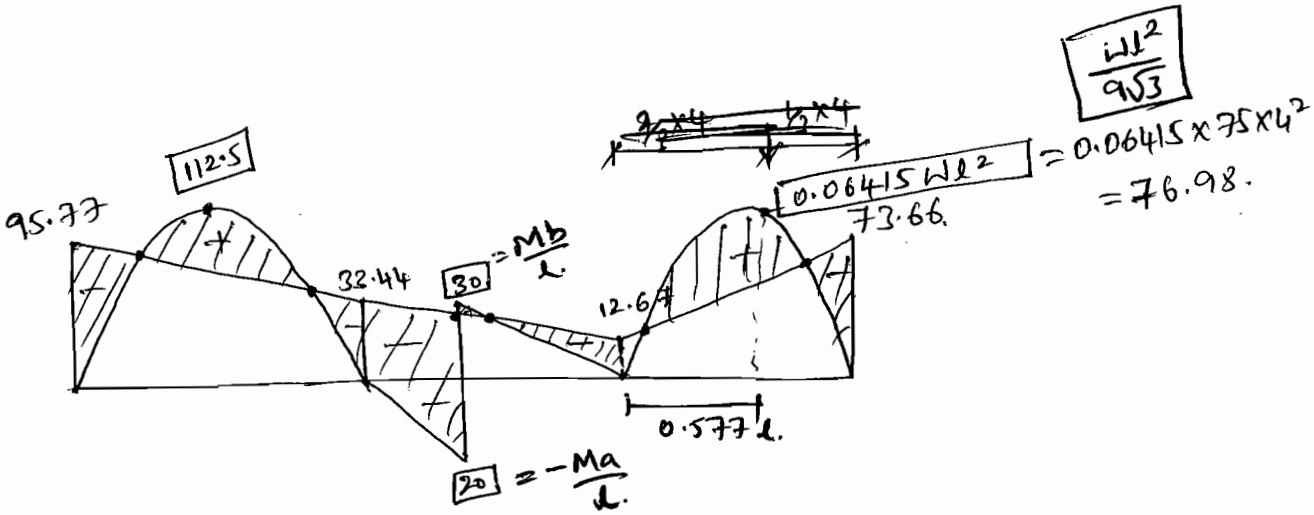
$M_{BA} = 33.44$ kN-m

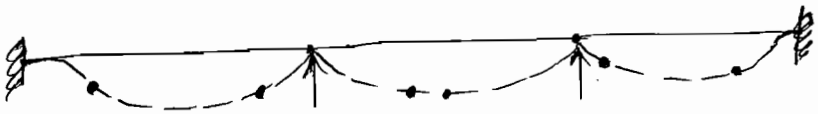
$M_{BC} = -33.44$ kN-m

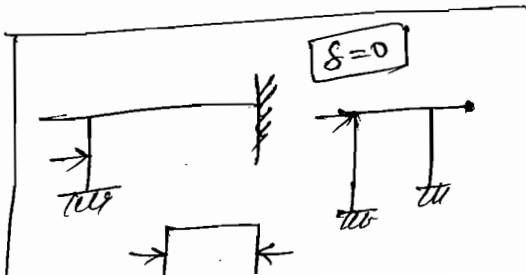
$M_{CB} = 12.67$ kN-m

$M_{CD} = -12.67$ kN-m

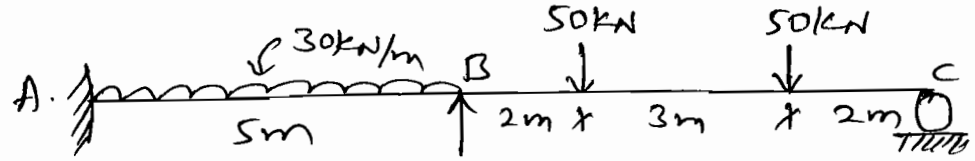
$M_{DC} = 73.66$ kN-m







(*) Analyze the beam shown in figure. The support A yields (or) rotates by $1/250$ radians. Take $EI = 5000\,kN\text{-}m^2$



Sol:

① FEM.

$$M_{FAB} = -\frac{wl^2}{12} = -62.5\,kN\text{-}m$$

$$M_{FBA} = \frac{wl^2}{12} = 62.5\,kN\text{-}m$$

$$M_{FBC} = -\frac{wab^2}{l^2} = -\frac{50\times2\times5^2}{7^2} - \frac{50\times5\times2^2}{7^2} = -71.42\,kN\text{-}m$$

$$M_{FCB} = -\frac{wa^2b}{l^2} = +\frac{50\times2^2\times5^2}{7^2} + \frac{50\times5^2\times2}{7^2} = 71.42\,kN\text{-}m$$

② S-D Eqn's

$$\theta_A = 1/250 \quad , \quad \delta = 0.$$

$$M_{AB} = -62.5 + \frac{2(5000)}{5}\left[(2\times1/250) + \theta_B\right]$$

$$= 2000\theta_B - 46.5 \quad \text{——— (1)}$$

$$M_{BA} = 62.5 + \frac{2(5000)}{5}\left[(2\times\theta_B) + 1/250\right]$$

$$= 4000\,\theta_B + 70.5 \quad \text{——— (2)}$$

$$M_{BC} = -71.42 + \frac{2(5000)}{7}\left[2\theta_B + \theta_C\right]$$

$$= 2857.14\,\theta_B + 1428.5\,\theta_C - 71.42 \quad \text{——— (3)}$$

$$M_{CB} = +71.42 + \frac{2(5000)}{7}\left[2\theta_C + \theta_B\right]$$

$$= 1428.5\theta_B + 2857.14\theta_C + 71.42 \quad \text{—— (4)}$$

(3) J. E. E.

Joint B, $M_{BA} + M_{BC} = 0.$

$$4000\theta_B + 70.5 + 2857.14\theta_B + 1428.5\theta_C - 71.42 = 0.$$

$$6857.14\theta_B + 1428.5\theta_C = 0.92 \quad \text{—— (I)}$$

Joint C, $M_{CB} = 0.$

$$1428.5\theta_B + 2857.14\theta_C = -71.42 \quad \text{—— (II)}$$

Solving equ's (I) & (II)

$$\boxed{\begin{aligned}\theta_B &= 5.962\times10^{-3}\\ \theta_C &= -0.0279\end{aligned}}$$

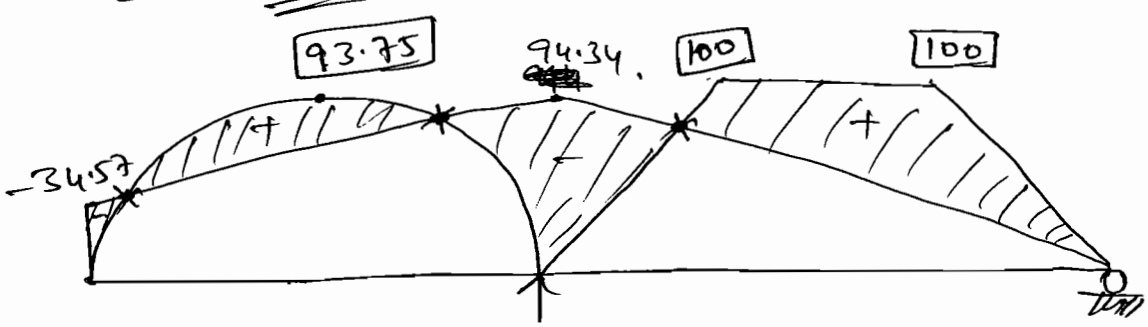
(4) Final Moments.

$M_{AB} = -34.576$ kNm

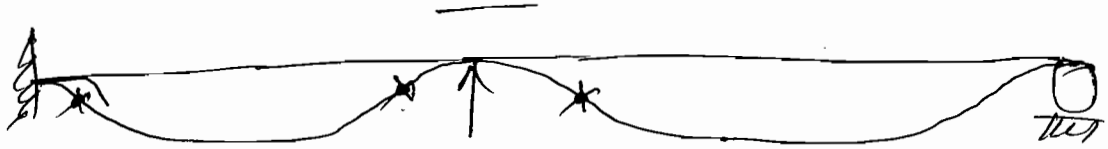
$M_{BA} = 94.34$ kN-m

$M_{BC} = -94.3$ kN-m

$M_{CB} = 0.$

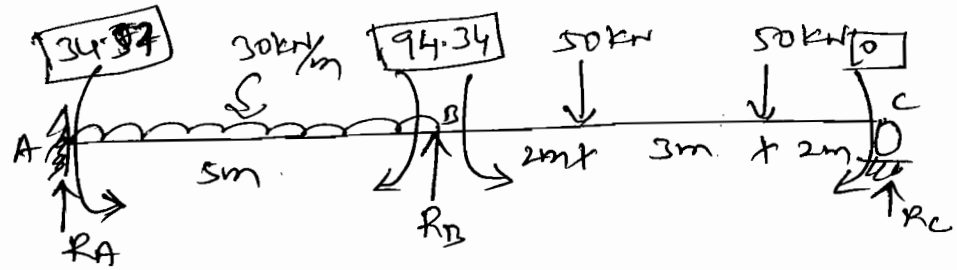


BMD.



EC.

SFD



$\Sigma M_B = 0$ (LHS)

$$(R_A \times 5) - 34.57 + 94.34 - \left(30 \times 5 \times \frac{5}{2}\right) = 0.$$

$$R_A = 63.04 \text{ kN.}$$

$\Sigma M_B = 0$ (RHS)

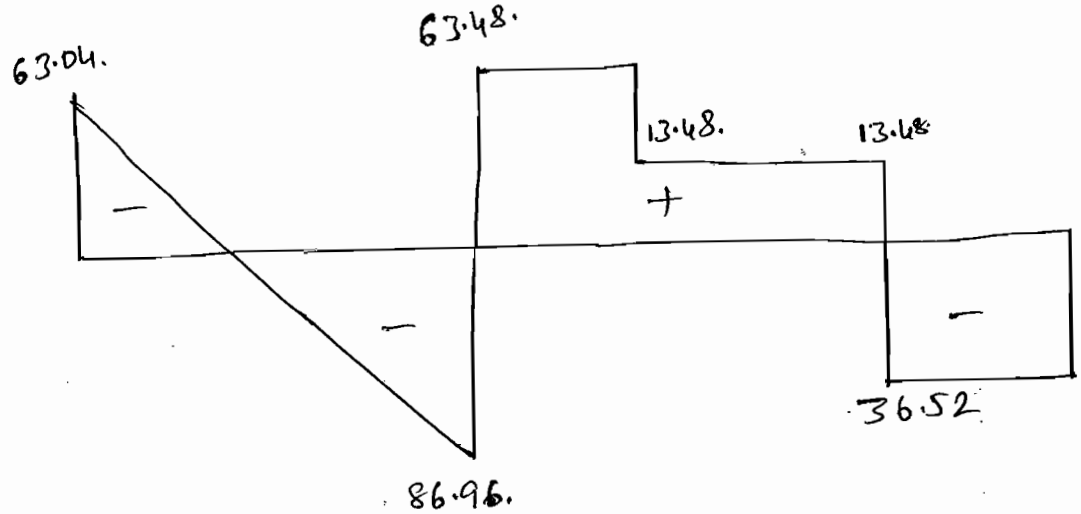
$$(-R_C \times 7) + (50 \times 5) + (50 \times 2) - 94.34 = 0.$$

$$R_C = 36.52 \text{ kN.}$$

$\Sigma V = 0.$

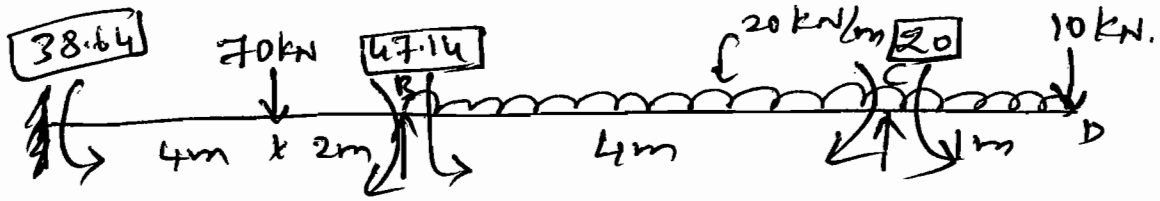
$$R_A + R_B + R_C = (30 \times 5) + (50) + (50)$$

$$R_B = 100.44 \text{ kN.}$$



SFD.

SFD for Problem (1) [illegible]



$\Sigma V = 0.$

$R_A + R_B + R_C = 70 + (20 \times 5) + 10 = 180$ — (1)

$\Sigma M_B = 0.$ (LHS)

$(R_A \times 6) - 38.64 + 47.14 - (70 \times 2) = 0.$

$\boxed{R_A = 21.91 \text{ kN}}$

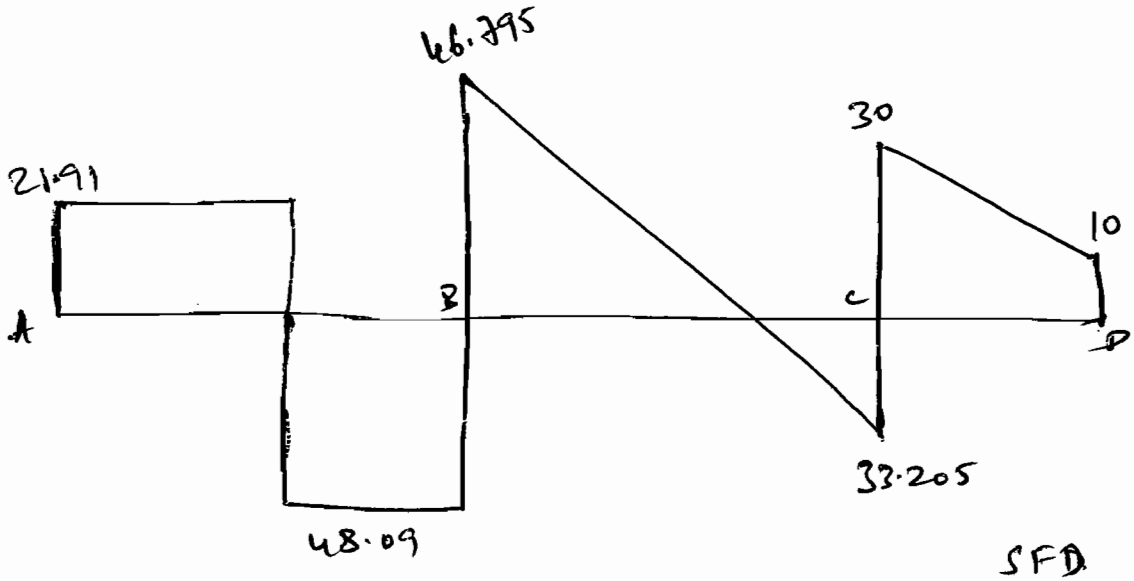
$\Sigma M_B = 0$ (RHS)

$-47.14 + 20 - 20 + (20 \times 5 \times \frac{5}{2}) + (10 \times 5) - (R_C \times 4) = 0$

$\boxed{R_C = 63.205 \text{ kN}}$

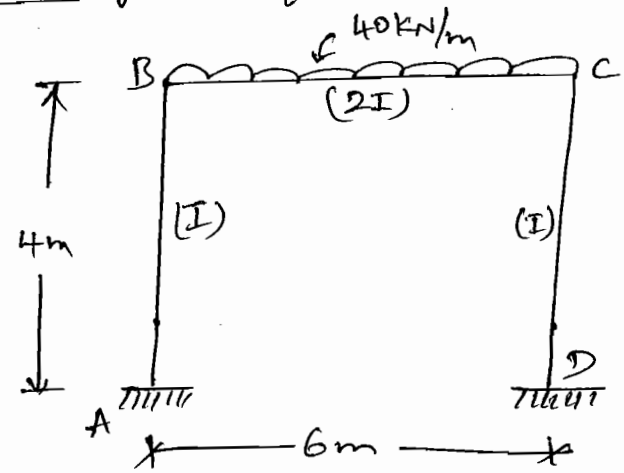
(1) $\Rightarrow R_B = 180 - (21.91 + 63.205)$

$\boxed{R_B = \cancel{63.215} \text{ kN}}$ 94.885



SFD

1) Analyse the given frame, draw BMD & EC.



Sol:

① FEM.

$M_{FAB} = M_{FBA} = M_{FCD} = M_{FDC} = 0.$

$M_{FBC} = -\frac{Wl^2}{12} = -\frac{40\times 6^2}{12} = -120\,kN\text{-}m$

$M_{FCB} = \frac{40\times 6^2}{12} = 120\,kN\text{-}m$

② S-D Eqns.

$M_{AB} = 0 + \frac{2EI}{4}\left(2\cancel{\theta_A}^{0} + \theta_B - \cancel{\frac{3\Delta}{L}}^{0}\right) = 0.5EI\theta_B$ —— ①

$M_{BA} = 0 + \frac{2EI}{4}(2\theta_B) = EI\theta_B$ —— ②

$M_{BC} = -120 + \frac{2E(2I)}{6}(2\theta_B + \theta_C) = -120 + \frac{4}{3}EI\theta_B + \frac{2}{3}EI\theta_C$ —— ③

$M_{CB} = 120 + \frac{2E(2I)}{6}(2\theta_C + \theta_B) = 120 + \frac{2}{3}EI\theta_B + \frac{4}{3}EI\theta_C$ —— ④

$M_{CD} = 0 + \frac{2EI}{4}(2\theta_C) = EI\theta_C$ —— ⑤

$M_{DC} = 0 + \frac{2EI}{4}(\theta_C) = 0.5EI\theta_C$ —— ⑥.

③ Equm Condition

$\Sigma M_B = 0.$, $M_{BA} + M_{BC} = 0$

$EI\theta_B - 120 + \frac{4}{3}EI\theta_B + \frac{2}{3}EI\theta_C = 0.$

Joint C, $M_{CB} + M_{CD} = 0$

$$120 + 0.66EI\theta_B + 1.33EI\theta_C + EI\theta_C = 0$$

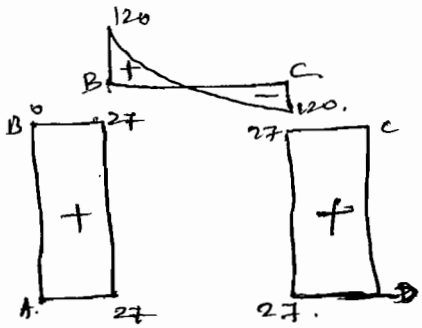
$$0.66EI\theta_B + 2.33EI\theta_C = -120 \quad \text{—— (8)}$$

Solving (7) & (8)

$$EI\theta_B = +71.85 \simeq \underline{72}.$$

$$EI\theta_C = \underline{-72}$$

SFD.



(4) Final Moments.

$$M_{AB} = 0.5 \times 72 = \underline{36\,kN\text{-}m}$$

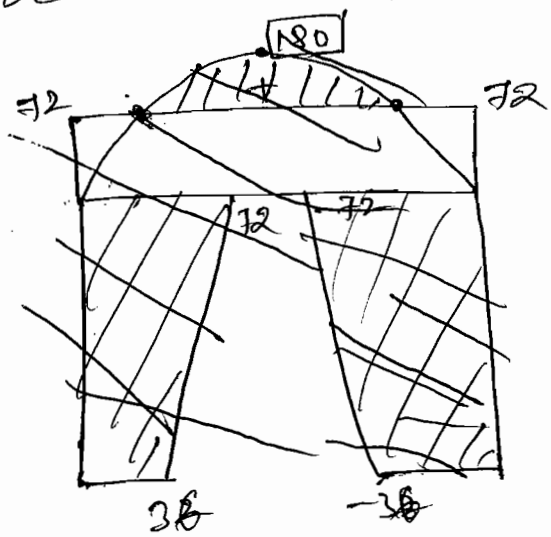
$$M_{BA} = \underline{72\,kN\text{-}m}$$

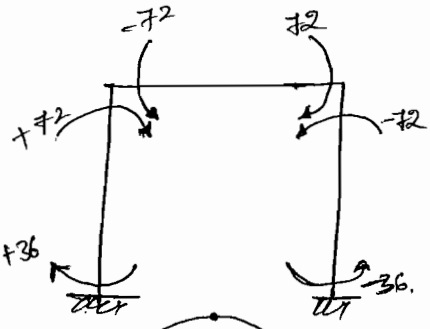
$$M_{BC} = -120 + \tfrac{4}{3}(72) + \tfrac{2}{3}(-72) = -71.76 \simeq \underline{-72\,kN\text{-}m}$$

$$M_{CB} = 120 + \tfrac{2}{3}(72) + \tfrac{4}{3}(-72) = 71.76 \simeq \underline{72\,kN\text{-}m}$$

$$M_{CD} = \underline{-72\,kN\text{-}m}$$

$$M_{DC} = 0.5(-72) = \underline{-36\,kN\text{-}m}$$





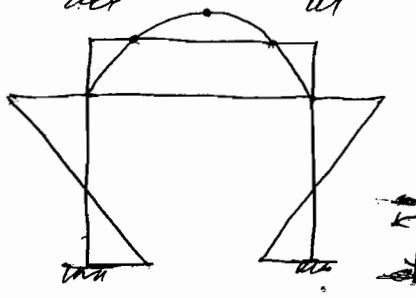
BM

BC,

$$= \frac{Wl^2}{8}$$

$$= \frac{40 \times 6^2}{8}$$

$$= \underline{180\,kN\text{-}m}$$



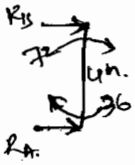
$\Sigma M_C = 0.$

$+R_D \times 4 - 72 - 36 =$

$R_D = \underline{27\,kN}.$

$\Sigma H = 0.$

$R_C = \underline{-27\,kN}.$



$\Sigma M_B = 0.$

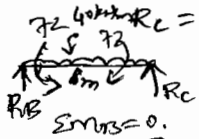
$-R_A \times 4 + 36 + 72 = 0$

$R_A = \underline{27\,kN}$

$\Sigma H = 0.$

$R_A + R_B = 0.$

$R_B = \underline{-27\,kN}$



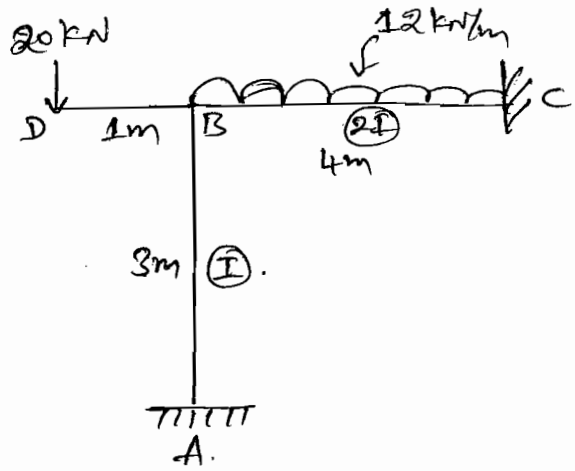
$\Sigma V = 0.$

$R_B + R_C = 40 \times 6.$

$\Sigma M_B = 0.$

$-R_C \times 6 + 72 - 72 + (40 \times 6 \times 6/2) = 0.$

$R_C = \underline{120\,kN}.$



Sol:-

① FEM.

$$M_{FAB} = M_{FDA} = 0.$$

$$M_{FBC} = \frac{-Wl^2}{12} = \frac{-12\times4^2}{12} = -16\ kN\text{-}m$$

$$M_{FCB} = \frac{Wl^2}{12} = 16\ kN\text{-}m$$

$$M_{BD} = 20\times1 = 20\ kN\text{-}m$$

② S-D equs. $\theta_A = 0,\ \theta_C = 0,\ \Delta = 0.$

$$M_{AB} = 0 + \frac{2EI}{3}(\theta_B) = \frac{2}{3}EI\theta_B \quad \text{——— (1)}$$

$$M_{BA} = \frac{2}{3}EI(2\theta_B) = 1.33\,EI\theta_B \quad \text{——— (2)}$$

$$M_{BC} = -16 + \frac{2E(2I)}{4}(2\theta_B + \cancel{\theta_C}^{\,0})$$

$$= -16 + 2EI\theta_B + \cancel{EI\theta_C} \quad \text{——— (3)}$$

$$M_{CB} = 16 + \frac{2E(2I)}{4}(\theta_B + \cancel{2\theta_C}^{\,0})$$

$$= 16 + EI\theta_B + \cancel{2EI\theta_C} \quad \text{——— (4).}$$

③ Equm Equ's.

At joint B, $\Sigma M_{BA} + M_{BC} + M_{BD} = 0$

$$1.33EI\theta_B + 2EI\theta_B + EI\theta_C - 16 + 20 = 0$$

$$EI\theta_B = -1.2$$

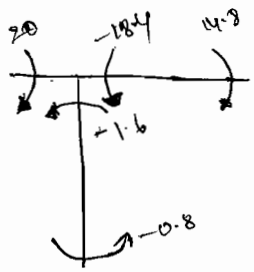
④ Final Moments.

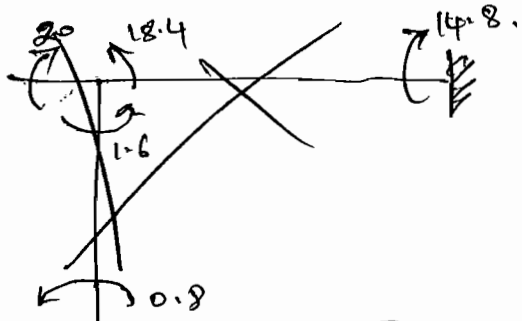
$$M_{AB} = \tfrac{2}{3}(-1.2) = -0.792 \simeq -0.8 \text{ kN-m}$$

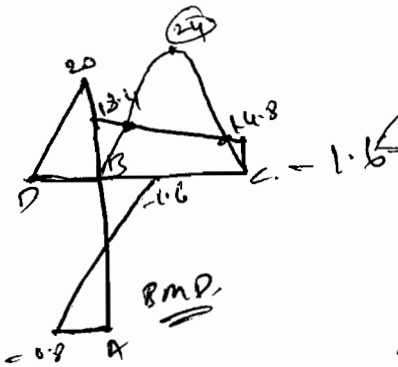
$$M_{BA} = \tfrac{4}{3}(-1.2) = -1.6 \text{ kN-m}$$

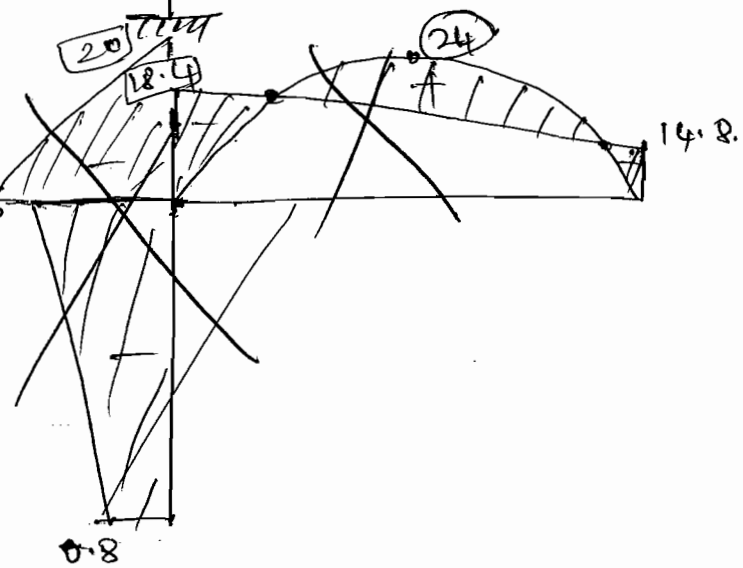
$$M_{BC} = -16 + \tfrac{2}{3}(-1.2) = -18.4 \text{ kN-m}$$

$$M_{CB} = 16 + (-1.2) = 14.8 \text{ kN-m}$$





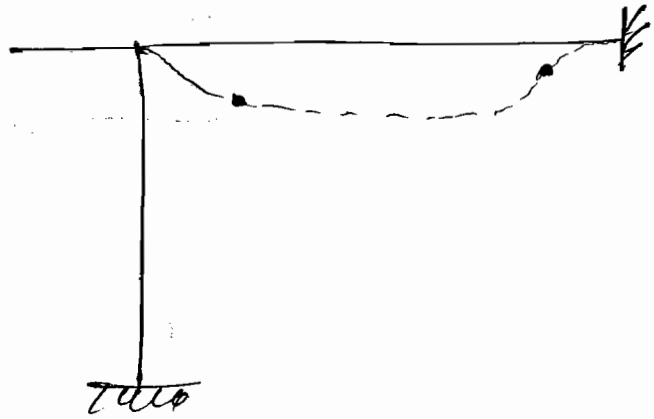




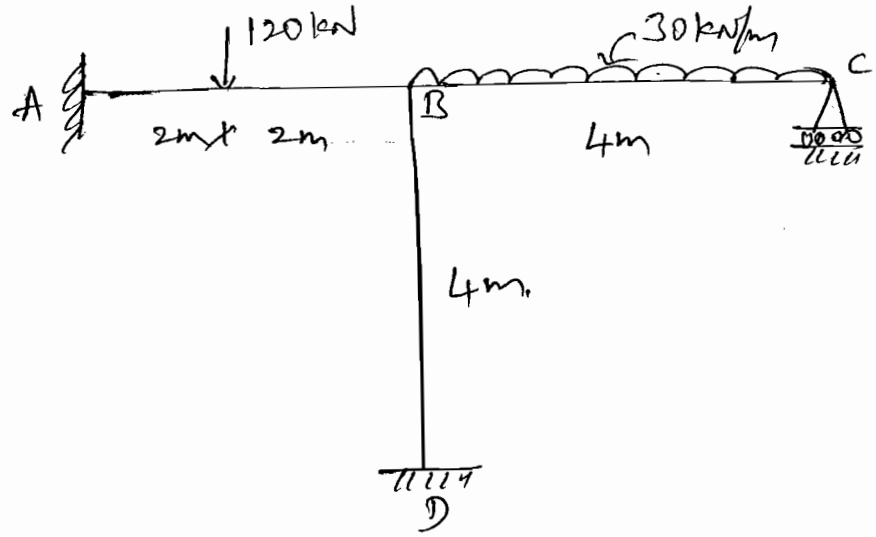
BC

$$\frac{Wl^2}{8} = \frac{+12\times4^2}{8}$$

$$= 24 \text{ kN-m}$$



(3) Analyse the frame shown in fig, using S.D.M



Sol:-

(1) FEM.

$M_{FBD} = M_{FDB} = 0.$

$M_{FAB} = \frac{-Wl}{8} = \frac{-120 \times 4}{8} = -60 \text{ kN-m}$

$M_{FBA} = \frac{Wl}{8} = 60 \text{ kNm.}$

$M_{FBC} = \frac{-Wl^2}{12} = \frac{-30 \times 4^2}{12} = -40 \text{ kN-m}$

$M_{FCB} = \frac{Wl^2}{12} = 40 \text{ kN-m}$

(2) S-D. Equ. $\theta_A = 0, \ \theta_D = 0, \ \Delta = 0.$

$M_{AB} = -60 + \frac{2EI}{4}(\theta_B) = -60 + 0.5EI\theta_B$ —— (1)

$M_{BA} = 60 + \frac{2EI}{4}(2\theta_B) = 60 + EI\theta_B$ —— (2)

$M_{BC} = -40 + \frac{2EI}{4}(2\theta_B + \theta_C) = -40 + EI\theta_B + 0.5EI\theta_C$ —— (3)

$M_{CB} = 40 + \frac{2EI}{4}(\theta_B + 2\theta_C) = 40 + 0.5EI\theta_B + EI\theta_C$ —— (4)

$M_{BD} = \frac{2EI}{4}(2\theta_B) = EI\theta_B$ —— (5)

$M_{DB} = \frac{2EI}{4}(\theta_B) = 0.5EI\theta_B$ —— (6)

Joint B, $M_{BA} + M_{BC} + M_{BD} = 0.$

$$60 + EI\theta_B - 40 + EI\theta_B + 0.5EI\theta_C + EI\theta_B = 0$$

$$3EI\theta_B + 0.5EI\theta_C = -20 \quad \text{——} \quad (7)$$

Joint C, $M_{CB} + M_{CD} = 0.$

$$40 + 0.5EI\theta_B + EI\theta_C = 0.$$

$$0.5EI\theta_B + EI\theta_C = -40 \quad \text{——} \quad (8)$$

Solving (7) & (8),

$EI\theta_B = 0$ $\qquad$ $EI\theta_C = -40.$

(4) Final Moments.

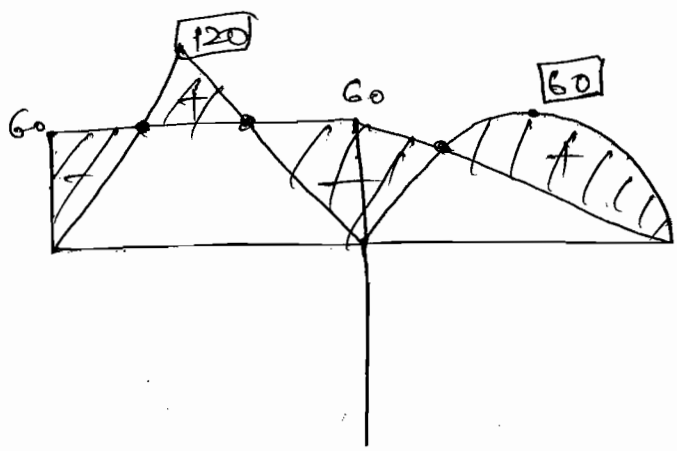
$M_{AB} = -60 + 0.5(0) = -60$ kN-m

$M_{BA} = 60 + 0 = 60$ kN-m

$M_{BC} = -40 + 0 + 0.5(-40) = -60$ kN-m

$M_{CB} = +40 + 0.5(0) + (-40) = 0$

$M_{BD} = 0$ $\qquad$ $M_{DB} = 0.$



BM

$AB = \frac{Wl}{4}$

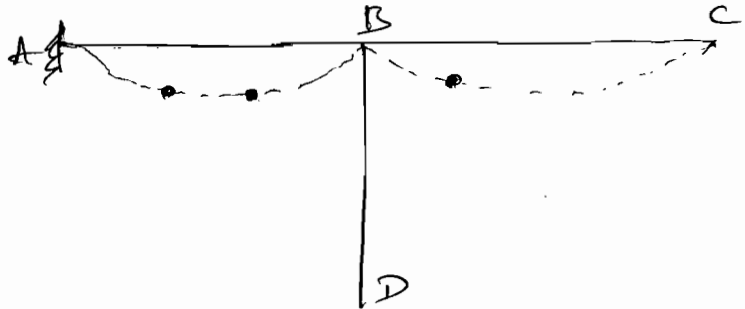
$= \frac{120 \times 4}{4}$

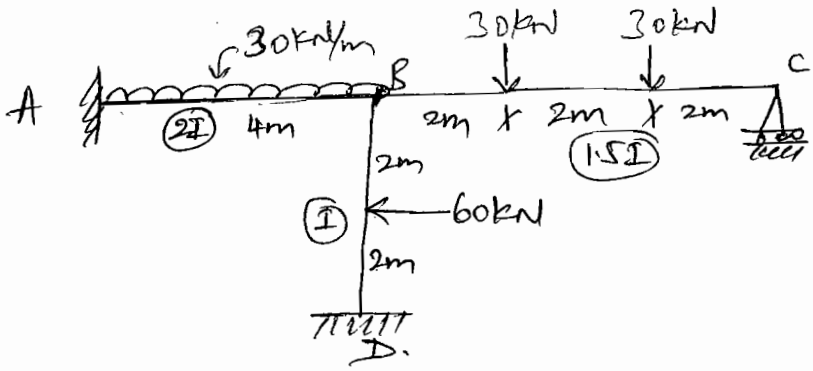
$= 120$ kN-m

$BC = \frac{Wl^2}{8}$

$= \frac{30 \times 4^2}{8}$

$= 60$ kN-m





Sol:- ① FEM.

$$M_{FAB} = -\frac{Wl^2}{12} = \frac{-30\times4^2}{12} = -40\,kN\text{-}m$$

$$M_{FAB} = +\frac{Wl^2}{12} = 40\,kN\text{-}m$$

$$M_{FBC} = -\left[\frac{Wab^2}{l^2} + \frac{Wab^2}{l^2}\right] = -\left[\frac{30\times2\times4^2}{6^2} + \frac{30\times4\times2^2}{6^2}\right] = -40\,kN\text{-}m$$

$$M_{FCB} = \left[\frac{Wa^2b}{l^2} + \frac{Wa^2b}{l^2}\right] = 40\,kN\text{-}m$$

$$M_{FBD} = -\frac{Wl}{8} = \frac{-60\times4}{8} = -30\,kN\text{-}m$$

$$M_{FDB} = +\frac{Wl}{8} = \frac{60\times4}{8} = 30\,kN\text{-}m$$

② S-D Equs.

$\theta_A = 0,\ \theta_D = 0,\ \Delta = 0.$

$$M_{AB} = -40 + \frac{2E(2I)}{4}(\theta_B) = -40 + EI\theta_B \quad \text{—— ①}$$

$$M_{BA} = 40 + \frac{2E(2I)}{4}(2\theta_B) = 40 + 2EI\theta_B \quad \text{—— ②}$$

$$M_{BC} = -40 + \frac{2E(1.5I)}{6}(2\theta_B + \theta_C) = -40 + EI\theta_B + 0.5EI\theta_C \quad \text{—— ③}$$

$$M_{CB} = 40 + \frac{2E(1.5I)}{6}(2\theta_C + \theta_B) = 40 + EI\theta_C + 0.5EI\theta_B \quad \text{—— ④}$$

$$M_{BD} = -30 + \frac{2EI}{4}(2\theta_B) = -30 + EI\theta_B \quad \text{—— ⑤}$$

$$M_{DB} = 30 + \frac{2EI}{4}(\theta_B) = 30 + 0.5EI\theta_B. \quad \text{—— ⑥}$$

③ Joint Equm Equs.

At joint B, $M_{BA} + M_{BC} + M_{BD} = 0.$

$$40 + 2EI\theta_B - 40 + EI\theta_B + 0.5EI\theta_C - 30 + EI\theta_B = 0.$$

$$4EI\theta_B + 0.5EI\theta_C = 30 \quad \text{—— ⑦}$$

$$40 + EI\theta_C + 0.5EI\theta_B = 0$$

$$0.5EI\theta_B + EI\theta_C = -40. \quad \text{—— (8)}$$

Solving (7) & (8)

$EI\theta_B = 13.33.$ $\qquad$ $EI\theta_C = -46.67.$

(4) Final Moments.

$M_{AB} = -40 + 13.33 = -26.67\ kN\text{-}m$

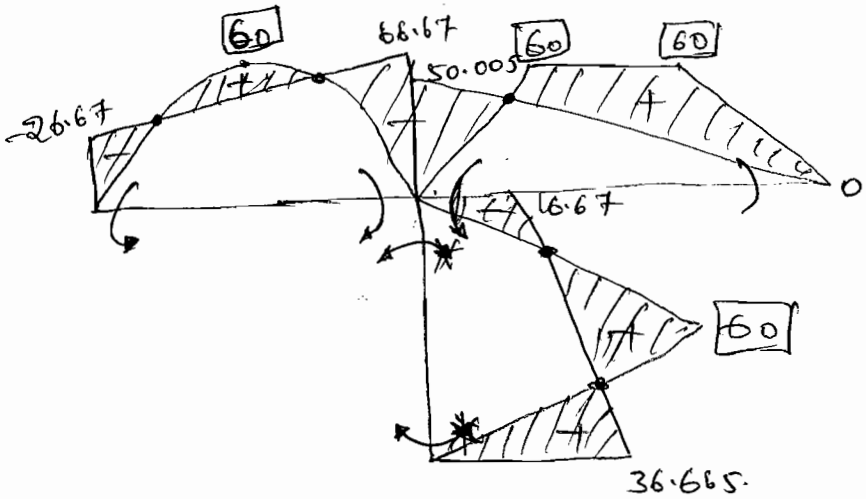
$M_{BA} = 40 + 13.33 = 66.67\ kN\text{-}m$

$M_{BC} = -40 + 13.33 + 0.5(-46.67) = -50.005\ kN\text{-}m$

$M_{CB} = 40 + 0.5(13.33) + (-46.67) = -5\times10^{-03} \simeq 0.$

$M_{BD} = -30 + 13.33 = -16.67\ kN\text{-}m$

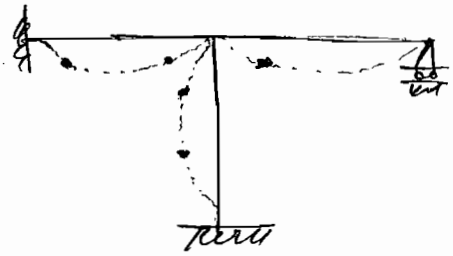
$M_{DB} = 30 + 13.33(0.5) = 36.665\ kN\text{-}m$



$BM)_{AB} = \frac{Wl^2}{8} = \frac{30\times4^2}{8} = 60\ kN\text{-}m$

$BM)_{BC} = \frac{Wl}{3} = \frac{30\times6}{2} = 60\ kN\text{-}m$

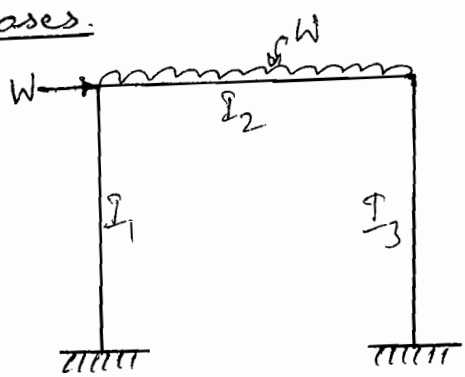
$BM)_{BD} = \frac{Wl}{4} = \frac{60\times4}{4} = 60\ kN\text{-}m$

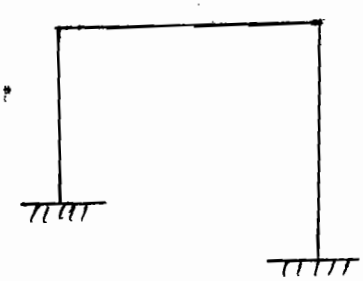


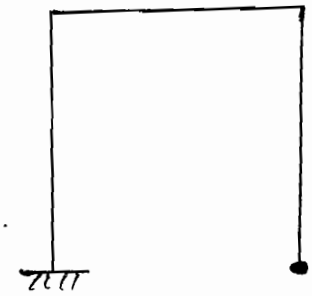
EC

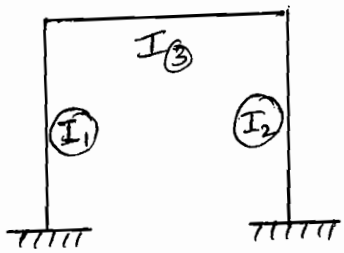
SWAY ANALYSIS - Slope Deflection Method.

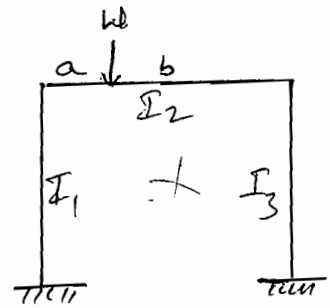
Cases.

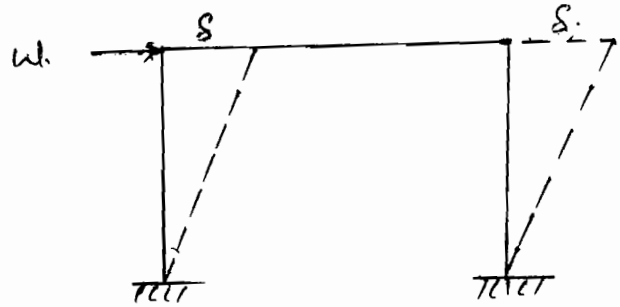












δ → Sway

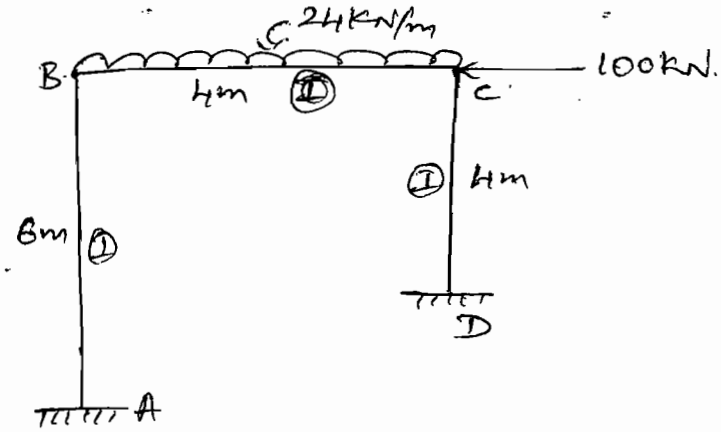
'δ' is an additional unknown

(i) 'δ' is taken only for Vertical members.

(ii) δ = 0, for Horizontal Members.

Problems.

1) Analyse the portal frame shown in fig by S-D method. Draw BMD & elastic Curve (EC)



Sol:-

(i) FEM.

$M_{FAB} = M_{FBA} = 0.$

$M_{FCD} = M_{FDC} = 0$

$M_{FBC} = \frac{-wl^2}{12} = \frac{-24 \times 4^2}{12} = -32 \text{ kN-m}$

$M_{FCB} = \frac{wl^2}{12} = 32 \text{ kN-m}$

(ii) S-D Equation

$\theta_A = 0, \ \theta_D = 0.$

'δ' is an additional Unknown & is taken only for vertical members.

$M_{AB} = M_{FAB} + \frac{2EI}{l}\left[2\theta_A + \theta_B - \frac{3\Delta}{l}\right]$

$= 0 + \frac{2EI}{6}\left[\theta_B - \frac{3\delta}{6}\right]$

$= 0.33 EI\theta_B - 0.166 EI\delta. \longrightarrow (1)$

$M_{BA} = 0 + \frac{2EI}{6}\left[2\theta_B - \frac{3\delta}{6}\right]$

$= 0.66 EI\theta_B - 0.166 EI\delta. \longrightarrow (2)$

$$M_{BC} = \frac{2EI}{4}\left[2\theta_B + \theta_C\right] - 32$$

$$= EI\theta_B + 0.5EI\theta_C - 32 \longrightarrow ③.$$

$$M_{CB} = \frac{2EI}{4}\left[\theta_B + 2\theta_C\right] + 32$$

$$= 0.5EI\theta_B + EI\theta_C + 32 \longrightarrow ④.$$

$$M_{CD} = \frac{2EI}{4}\left[2\theta_C - \frac{3\delta}{4}\right]$$

$$= EI\theta_C - 0.375EI\delta. \longrightarrow ⑤$$

$$M_{DC} = \frac{2EI}{4}\left[\theta_C - \frac{3\delta}{4}\right]$$

$$= 0.5EI\theta_C - 0.375EI\delta \longrightarrow ⑥.$$

(iii) Equilibrium Condition.

① At joint B, $M_{BA} + M_{BC} = 0.$

$$1.66EI\theta_B + 0.5EI\theta_C - 0.166EI\delta = 32 \longrightarrow \text{I}.$$

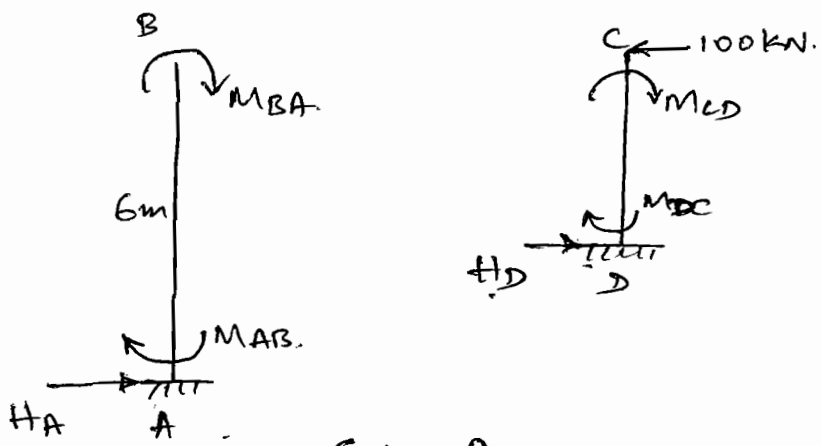
② At joint C, $M_{CB} + M_{CD} = 0.$

$$0.5EI\theta_B + 2EI\theta_C - 0.375EI\delta = -32 \longrightarrow \text{II}$$

③ Shear Condition

Consider only Vertical members with horizontal loads.

Assume all the members in clockwise direction



$$\Sigma H = 0$$

$$H_A + H_D = 100 \longrightarrow ⓐ.$$

$$\Sigma M_B = 0.$$

$$-H_A \times 6 + M_{AB} + M_{BA} = 0.$$

$$H_A = \frac{1}{6}\left[M_{AB} + M_{BA}\right]$$

$$= \frac{1}{6}\left[0.33EI\theta_B - 0.166EI\delta + 0.66EI\theta_B - 0.166EI\delta\right]$$

$$= \frac{1}{6}\left[EI\theta_B - 0.332EI\delta\right]$$

$$H_A = 0.166EI\theta_B - 0.055EI\delta \longrightarrow (b)$$

$\Sigma M_C = 0.$

$$-H_D \times 4 + M_{CD} + M_{DC} = 0$$

$$H_D = \frac{1}{4}\left[1.5EI\theta_C - 0.75EI\delta\right]$$

$$H_D = 0.375EI\theta_C - 0.1875EI\delta \longrightarrow (c)$$

Subs (b) & (c) in eqn (a).

$$0.166EI\theta_B - 0.055EI\delta + 0.375EI\theta_C - 0.1875EI\delta = 100$$

$$0.166EI\theta_B + 0.375EI\theta_C - 0.243EI\delta = 100 \longrightarrow (III)$$

Solving (I), (II), & (III)

$$\boxed{\theta_B = -2.89/EI. \qquad \delta = -615.03/EI. \qquad \theta_C = -130.60/EI.}$$

(iv) Final Moments.

$M_{AB} = 101.14$ kN-m

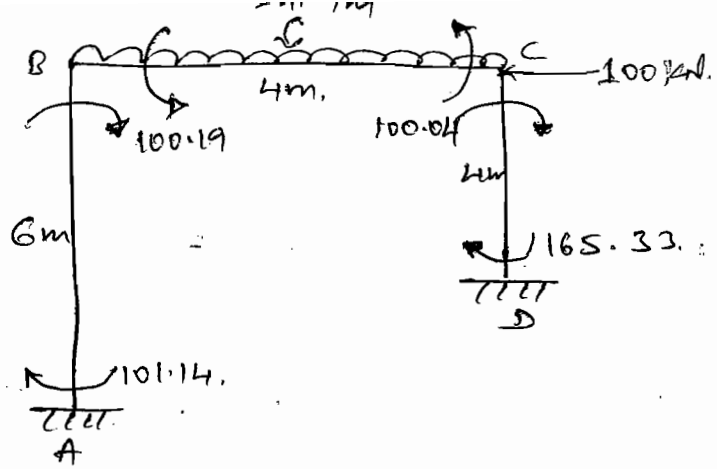
$M_{BA} = 100.18$ kN-m

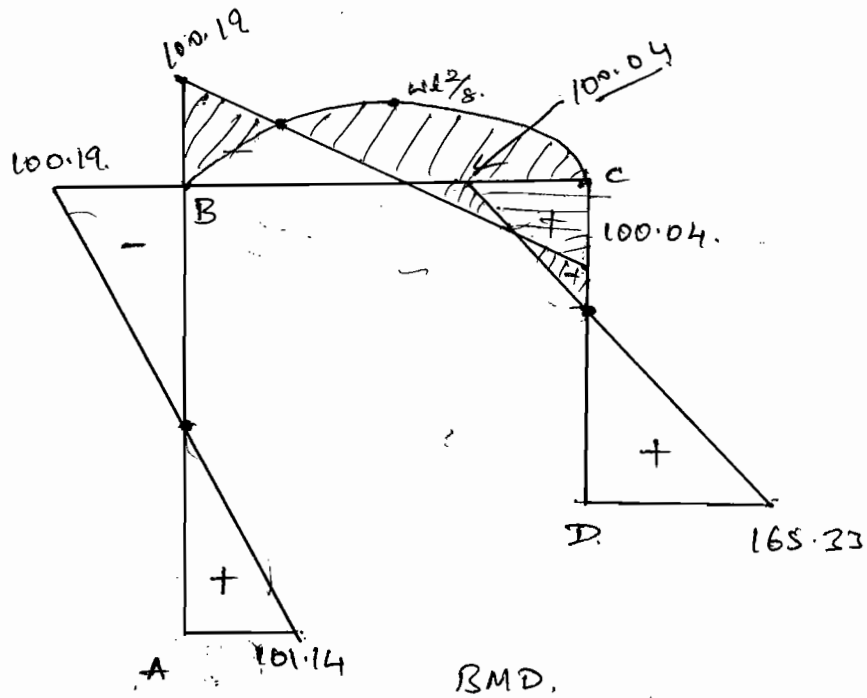
$M_{BC} = -100.19$ kN-m

$M_{CB} = -100.04$ kN-m

$M_{CD} = 100.04$ kN-m

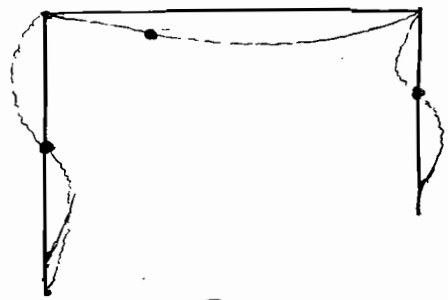
$M_{DC} = 165.33$ kN-m.





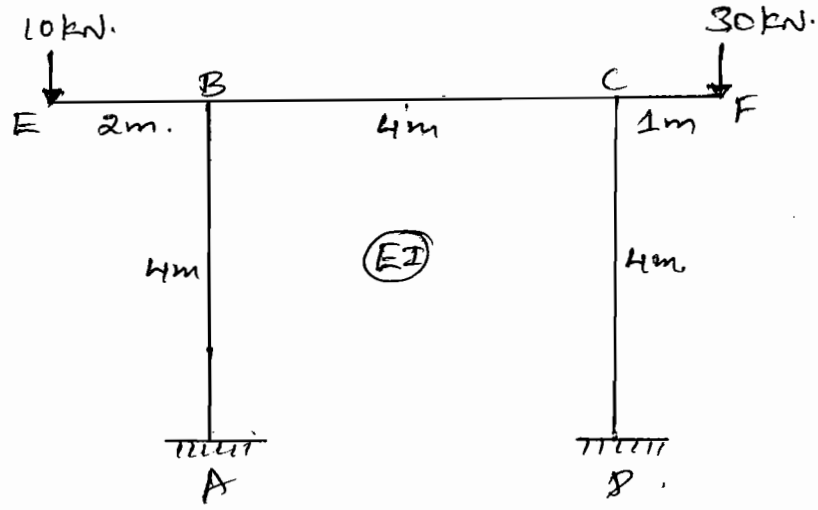
BMD.

$$BM)_{BC} = \frac{wl^2}{8} = \frac{24 \times 4^2}{8} = 48 \text{ kN-m}$$



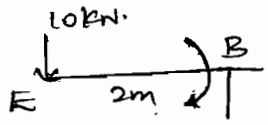
E.C.

(2). Analyse the portal frame by S-D method and draw BMD & EC.

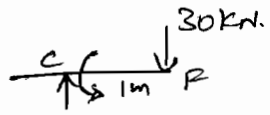


Sol:-

(i) FEM.



$M_{BE} = +10 \times 2 = 20\ kN\text{-}m$



$M_{CF} = -30 \times 1 = -30\ kN\text{-}m$

(ii) S.D. equations.

$\theta_A = 0, \quad \theta_D = 0.$

'δ' is an additional unknown & is taken only for Vertical Members.

There is no S-D equ for Overhanging portion.

$$M_{AB} = 0 + \frac{2EI}{4}\left[\theta_B - \frac{3\delta}{4}\right]$$

$$= 0.5EI\theta_B - 0.375EI\delta \longrightarrow (1)$$

$$M_{BA} = \frac{2EI}{4}\left[2\theta_B - \frac{3\delta}{4}\right]$$

$$= EI\theta_B - 0.375EI\delta \longrightarrow (2)$$

$$M_{BC} = \frac{2EI}{4}\left[2\theta_B + \theta_C\right]$$

$$= EI\theta_B + 0.5EI\theta_C \longrightarrow (3)$$

$$M_{CB} = \frac{2EI}{4}\left[2\theta_C + \theta_B\right]$$

$$= EI\theta_C + 0.5EI\theta_B \quad \text{—— (4)}$$

$$M_{CD} = \frac{2EI}{4}\left[2\theta_C - \frac{3\delta}{4}\right] = EI\theta_C - 0.375EI\delta \quad \text{—— (5)}$$

$$M_{DC} = \frac{2EI}{4}\left[\theta_C - \frac{3\delta}{4}\right] = 0.5EI\theta_C - 0.375EI\delta \quad \text{—— (6)}$$

(iii) <u>Equilibrium Eqn's</u>

@ joint B, $M_{BA} + M_{BC} + M_{BE} = 0.$

$$EI\theta_B - 0.375EI\delta + EI\theta_B + 0.5EI\theta_C + 20 = 0.$$

$$2EI\theta_B + 0.5EI\theta_C - 0.375EI\delta = -20 \quad \text{—— (I)}$$

@ joint C, $M_{CB} + M_{CD} + M_{CF} = 0.$

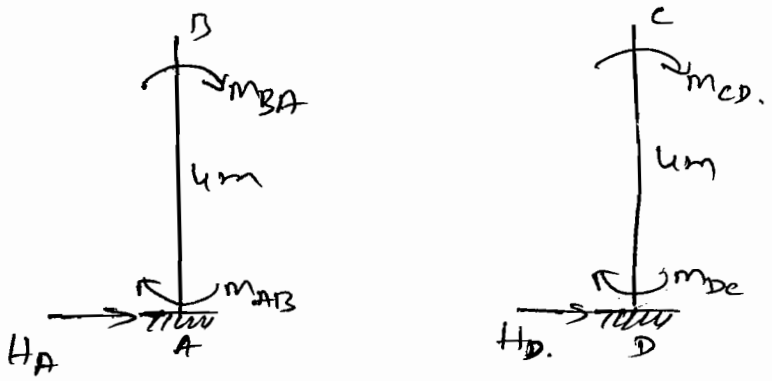
$$EI\theta_C + 0.5EI\theta_B + EI\theta_C - 0.375EI\delta - 30 = 0.$$

$$0.5EI\theta_B + 2EI\theta_C - 0.375EI\delta = 30 \quad \text{—— (II)}$$

(iv) <u>Shear Condition</u>

Consider only vertical members with <u>hori</u> loads.

Assume all the members in Clockwise direction.



$$\Sigma H = 0.$$

$$H_A + H_D = 0. \quad \text{—— (a)}$$

$$\Sigma M_B = 0$$

$$(-H_A \times 4) + M_{AB} + M_{BA} = 0.$$

$$H_A = \frac{0.5EI\theta_B - 0.375EI\delta + EI\theta_B - 0.375EI\delta}{4}.$$

$$H_A = 0.375EI\theta_B - 0.1875EI\delta \quad \text{—— (b)}$$

$$(-H_D \times 4) + M_{CD} + M_{DC} = 0.$$

$$H_D = \frac{EI\theta_C - 0.375EI\delta + 0.5EI\theta_C - 0.375EI\delta}{4}.$$

$$H_D = 0.375EI\theta_C - 0.1875EI\delta \quad \text{—— (C)}.$$

Subs (b) & (c) in equ (a).

$$0.375EI\theta_B + 0.375EI\theta_C - 0.375EI\delta = 0 \quad \text{—— (III)}$$

Solving equ (I), (II) & (III)

$$\boxed{\begin{aligned} EI\theta_B &= -13.80 \\ EI\theta_C &= 19.52 \\ EI\delta &= 5.71 \end{aligned}}$$

(V) Final Moments.

Subs the Values of $EI\theta_B$, $EI\theta_C$ & $EI\delta$ in equ's (1) to (6)

$M_{AB} = -9.04$ kN-m

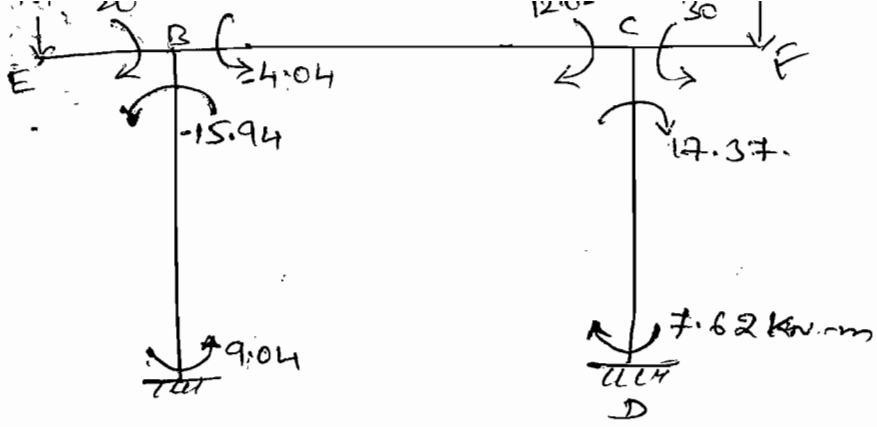
$M_{BA} = -15.94$ kN-m

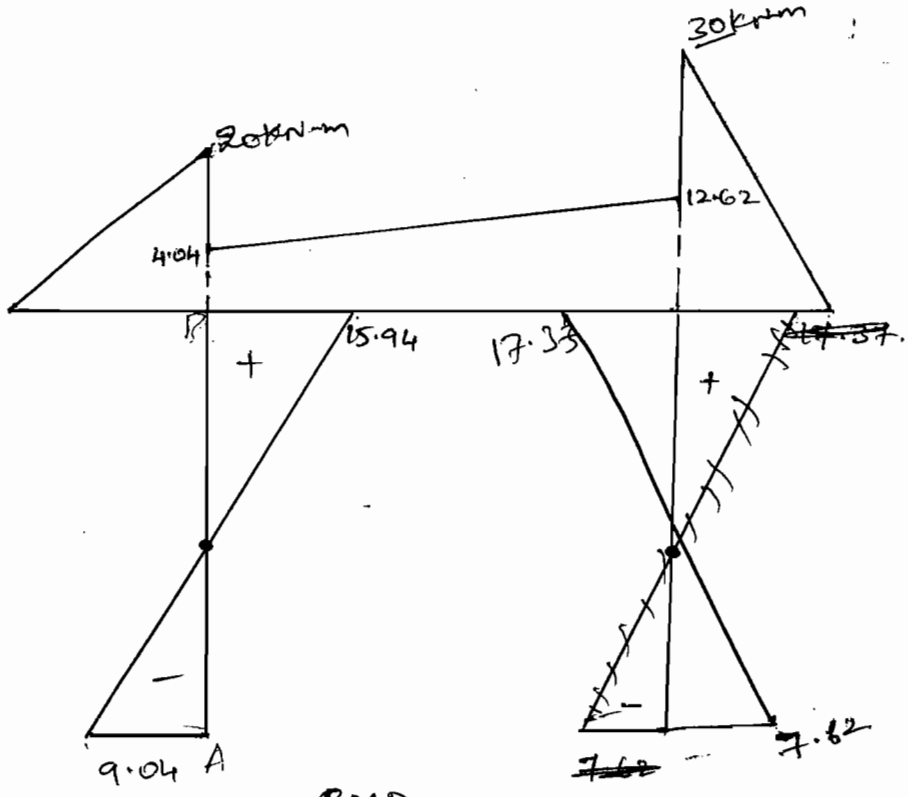
$M_{BC} = -4.04$ kN-m

$M_{CB} = 12.62$ kN-m

$M_{CD} = 17.37$ kN-m

$M_{DC} = 7.62$ kN-m


E
B
4.04
-15.94
9.04
C
30
F
17.37
7.62 kN.m
D

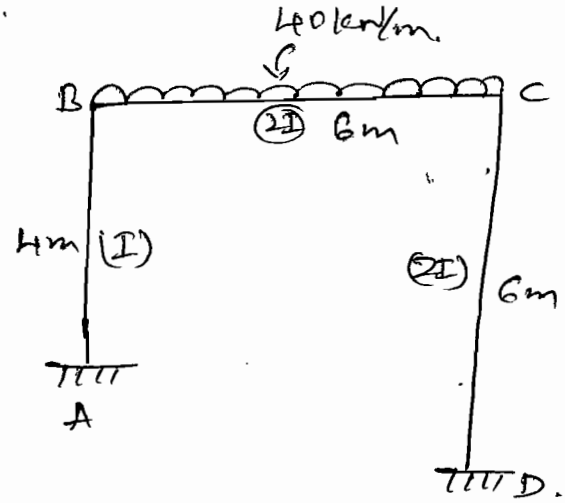

30kN-m
20kN-m
12.62
4.04
B
15.94
17.37
+
+
-
-
9.04
A
7.62

BMD.


B
E
C
F
A
D

EC.

3) Analyse the portal frame shown in figure by S-D method & Draw BMD & EC.



Sol:-

(i) FEM.

$$M_{FAB} = M_{FBA} = M_{FCD} = M_{FDC} = 0.$$

$$M_{FBC} = \frac{-wl^2}{12} = \frac{-40\times 6^2}{12} = -120\,kN\text{-}m$$

$$M_{FCB} = \frac{+wl^2}{12} = 120\,kN\text{-}m$$

(ii) S-D equation.

$$\theta_A = \theta_D = 0$$

'δ' is an additional ~~moment~~ unknown & is taken only for vertical members.

$$M_{AB} = 0 + \frac{2EI}{4}\left[\theta_B - \frac{3\delta}{4}\right]$$

$$= 0.5EI\theta_B - 0.375EI\delta \longrightarrow (1).$$

$$M_{BA} = 0 + \frac{2EI}{4}\left[2\theta_B - \frac{3}{4}\delta\right]$$

$$= EI\theta_B - 0.375EI\delta \longrightarrow (2)$$

$$M_{BC} = \frac{2E(2I)}{6}\left[2\theta_B + \theta_C\right] - 120.$$

$$= 1.33EI\theta_B + 0.667EI\theta_C - 120 \longrightarrow (3)$$

$$= 0.667EI\theta_B + 1.33EI\theta_C + 120 \longrightarrow (4)$$

$$M_{CD} = \frac{2E(2I)}{6}\left[2\theta_C - \frac{3\delta}{6}\right]$$

$$= 1.33EI\theta_C - 0.33EI\delta \longrightarrow (5)$$

$$M_{DC} = \frac{2E(2I)}{6}\left[\theta_C - \frac{3\delta}{6}\right]$$

$$= 0.66EI\theta_C - 0.33EI\delta \longrightarrow (6)$$

(iii) Equm Condition

(a) At Joint B.

$$M_{BA} + M_{BC} = 0.$$

$$2.33EI\theta_B + 0.667EI\theta_C - 0.375EI\delta = 120 \longrightarrow (I)$$

(b) At joint C

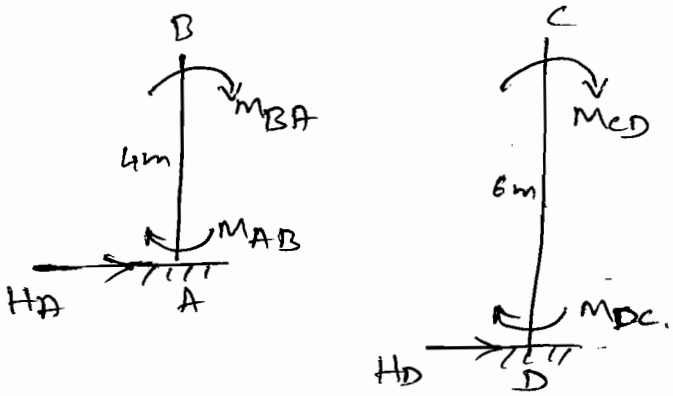
$$M_{CB} + M_{CD} = 0.$$

$$0.667EI\theta_B + 2.667EI\theta_C - 0.33EI\delta = -120 \longrightarrow (II)$$

(c) Shear Condition

Consider only vertical members with horizontal loads.

Assume all the moments ~~are~~ in clockwise direction.



$$\Sigma H = 0.$$

$$H_A + H_D = 0 \longrightarrow (a).$$

$\Sigma M_B = 0.$

$$-H_A \times 4 + M_{AB} + M_{BA} = 0.$$

$$H_A = \frac{1}{4}\left[1.5EI\theta_B - 0.75EI\delta\right]$$

$$H_A = 0.375EI\theta_B - 0.1875EI\delta \longrightarrow (b)$$

$\Sigma M_C = 0.$

$$-H_D \times 6 + M_{CD} + M_{DC} = 0$$

$$H_D = \frac{1}{6}\left[2EI\theta_C - 0.66EI\delta\right]$$

$$H_D = 0.33EI\theta_C - 0.11EI\delta \longrightarrow (c)$$

Subs (b) & (c) in (a).

$$0.375EI\theta_B + 0.33EI\theta_C - 0.2975EI\delta = 0. \longrightarrow (III).$$

Solving (I), (II) & (III).

$$\boxed{\begin{array}{ll} EI\theta_B = 72.73. & EI\delta = 25.02. \\ EI\theta_C = -60.08. & \end{array}}$$

(iv) Final Moments.

$M_{AB} = 26.98$ kN-m

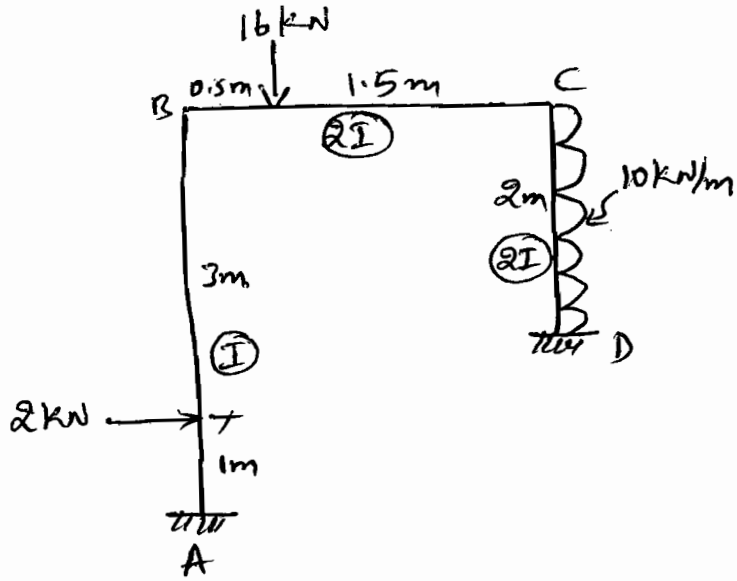
$M_{BA} = 63.34$ kN-m

$M_{BC} = -63.34$ kN-m

$M_{CB} = 88.60$ kN-m

$M_{CD} = -88.163$ kN-m

$M_{DC} = -47.90$ kN-m



Sol:- ① FEM.

$$M_{FAB} = -\frac{2\times 1\times 3^2}{4^2} = -1.125\ kNm$$

$$M_{FBA} = \frac{2\times 1^2\times 3}{4^2} = 0.375\ kNm$$

$$M_{FBC} = \frac{-16\times 0.5\times 1.5^2}{2^2} = -4.50\ kNm$$

$$M_{FCB} = \frac{16\times 0.5^2\times 1.5}{2^2} = 1.50\ kNm$$

$$M_{FCD} = \frac{-10\times 2^2}{12} = -3.33\ kNm$$

$$M_{FDC} = \frac{10\times 2^2}{12} = 3.33\ kNm$$

② S-D. Eqns

$$\theta_A = 0, \quad \theta_D = 0.$$

$$M_{AB} = -1.125 + \frac{2EI}{4}\left(\theta_B - \frac{3\Delta}{4}\right) = -1.125 + 0.5EI\theta_B - 0.375EI\Delta \quad (1)$$

$$M_{BA} = 0.375 + \frac{2EI}{4}\left(2\theta_B - \frac{3\Delta}{4}\right) = 0.375 + EI\theta_B - 0.375EI\Delta \quad (2)$$

$$M_{BC} = -4.5 + \frac{2E(2I)}{2}(2\theta_B + \theta_C) = -4.5 + 4EI\theta_B + 2EI\theta_C \quad (3)$$

$$M_{CB} = 1.5 + \frac{2E(2I)}{2}(2\theta_C + \theta_B) = 1.5 + 2EI\theta_B + 4EI\theta_C \quad (4)$$

$$M_{CD} = -3.33 + \frac{2E(2I)}{2}\left(2\theta_C - \frac{3\Delta}{2}\right) = -3.33 + 4EI\theta_C - 3EI\Delta \quad (5)$$

$$M_{DC} = 3.33 + \frac{2E(2I)}{2}\left(\theta_C - \frac{3\Delta}{2}\right) = 3.33 + 2EI\theta_C - 3EI\Delta \quad (6)$$

(3) J.E.E.

① Joint B, $M_{BA} + M_{BC} = 0$

$$0.375 + EI\theta_B - 0.375EI\Delta - 4.5 + 4EI\theta_B + 2EI\theta_C = 0.$$

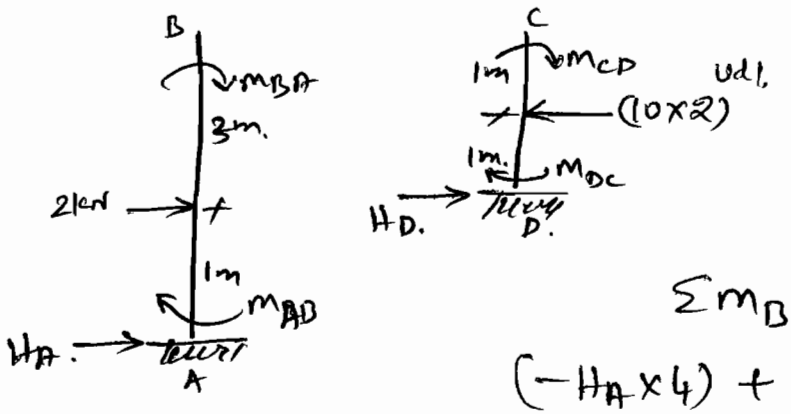
$$5EI\theta_B + 2EI\theta_C - 0.375EI\Delta = 4.125. \quad \text{— (I)}$$

② Joint C, $M_{CB} + M_{CD} = 0.$

$$1.5 + 2EI\theta_B + 4EI\theta_C - 3.333 + 4EI\theta_C - 3EI\Delta = 0.$$

$$2EI\theta_B + 8EI\theta_C - 3EI\Delta = 1.833 \quad \text{— (II)}.$$

③ Shear Condition



$\Sigma H = 0.$

$$H_A + H_D + 2 - (10\times2) = 0.$$

$$H_A + H_D = 18 \quad \text{— (a)}$$

$\Sigma M_B = 0.$

$$(-H_A \times 4) + M_{BA} + M_{AB} - (2\times3) = 0.$$

$$H_A = \frac{-1.125 + 0.5EI\theta_B - 0.375EI\Delta + 0.375 + EI\theta_B - 0.375EI\Delta - 6}{4.}$$

$$H_A = -1.6875 + 0.375EI\theta_B - 0.1875EI\Delta \quad \text{— (b)}.$$

$\Sigma M_C = 0.$

$$(-H_D \times 2) + M_{CD} + M_{DC} + (10\times2\times1) = 0.$$

$$H_D = \frac{M_{CD} + M_{DC} + 20.}{2.}$$

$$H_D = \frac{-3.33 + 4EI\theta_C - 3EI\Delta + 3.33 + 2EI\theta_C - 3EI\Delta + 20}{2.}$$

$$H_D = 10 + 3EI\theta_C - 3EI\Delta \quad \text{— (c)}$$

(a) ⇒ $0.375EI\theta_B + 3EI\theta_C - 3.1875EI\Delta = 9.68$ — (III)

Solving (I), (II) & (III).

$$\boxed{\begin{aligned} EI\theta_B &= 1.196 \\ EI\theta_C &= -1.78 \\ EI\Delta &= -4.57. \end{aligned}}$$

(4) Final Moments.

$M_{AB} = 1.16$ kN-m

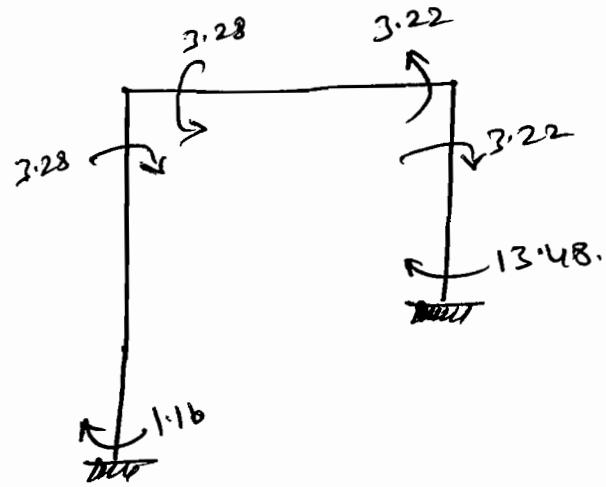
$M_{BA} = 3.28$ kN-m

$M_{BC} = -3.28$ kNm

$M_{CD} = -3.22$ kNm

$M_{CD} = 3.22$ kN-m

$M_{DC} = 13.48$ kNm

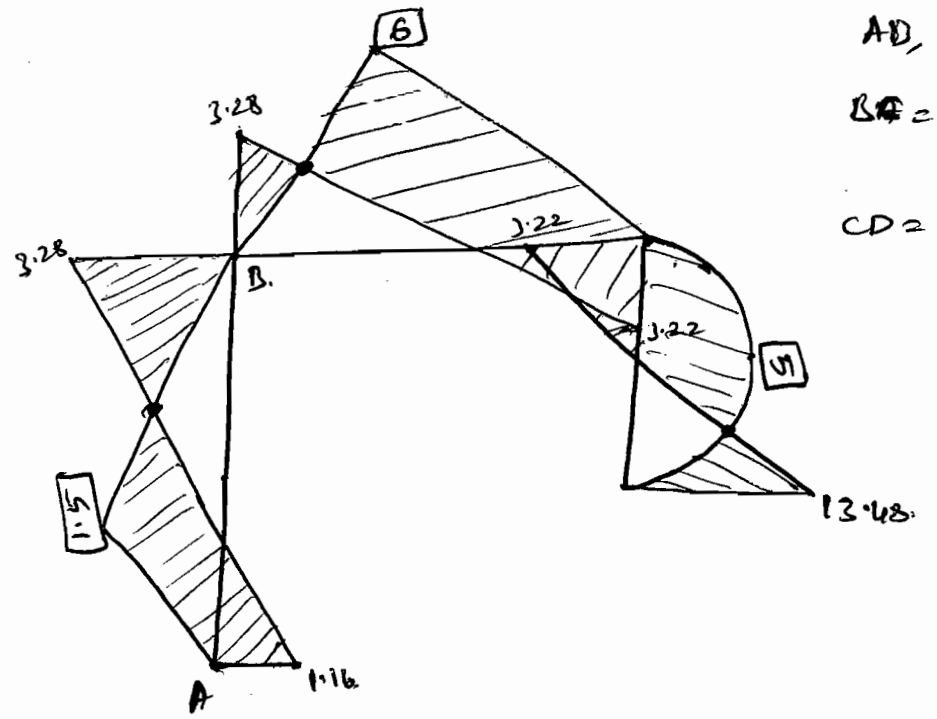


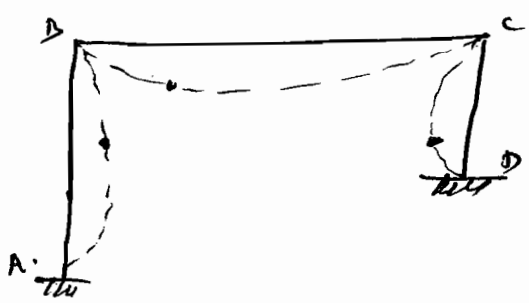
BM

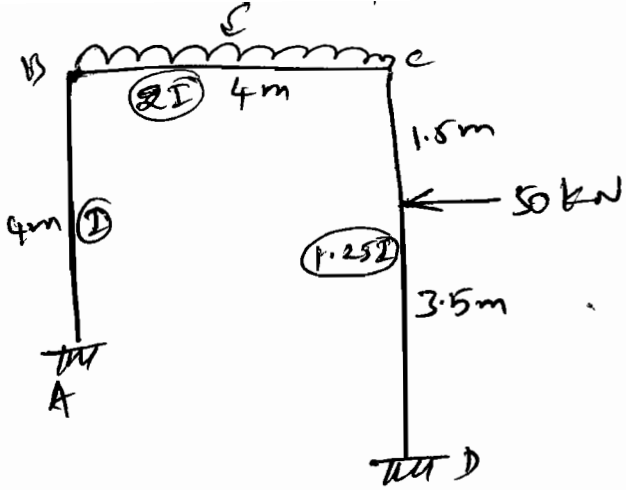
$AD = \frac{2 \times 1 \times 3}{4} = 1.5$ kNm

$BA = \frac{16 \times 0.5 \times 1.5}{2} = 6$ kNm

$CD = \frac{10 \times 2^2}{8} = 5$ kNm







-50.85 $+6.20$

-6.20

$+50.85$

$+52.12$ $+52.42$

$EI\theta_B = -2.63$, $EI\theta_C = -12.24$, $EI\Delta = -142.64$

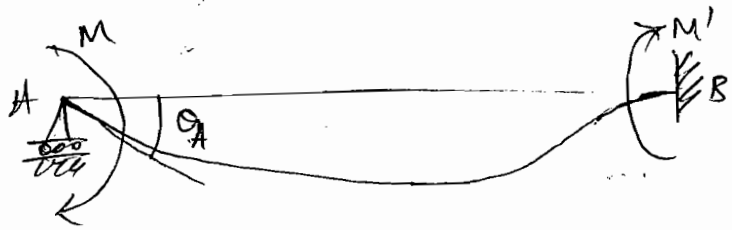
Introduction

This method is widely used for the analysis of indeterminate structures. In this method, solution of simultaneous equations of slope deflection method is replaced by an iterative distribution procedure. For fairly higher degree of indeterminate structures this method is ideally suited.

Terminology.

① Carry Over Moment.

When a moment is applied at one end of a member allowing rotation of that end & fixing the far end, some moment develops at the far end also, this moment is called Carry Over Moment.



Thus, in beam AB shown in figure, if M is the moment applied at end A, allowing rotation of A and M' is the moment developed at B, then M' is the Carry over Moment.

② Carry Over factor

The ratio of Carry over moment to applied moment is called Carry over factor.

In beam AB,

$$\text{Carry over factor} = \frac{M'}{M}$$

③ Stiffness

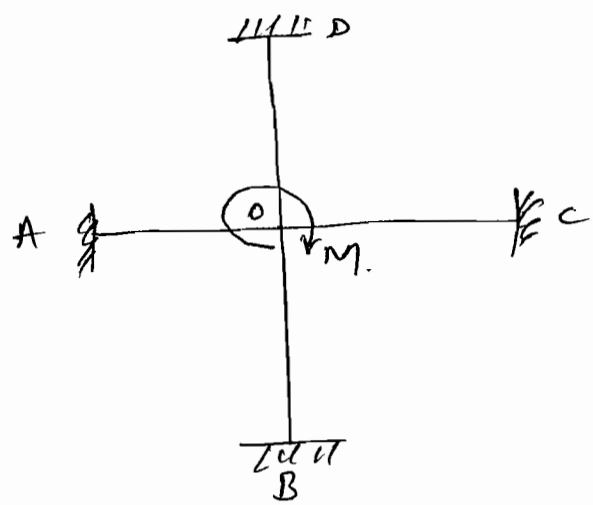
Moment required to rotate an end by unit angle (1 radian), when rotation is permitted at that end, is called stiffness of the beam. Thus in the beam AB, if θ_A is the rotation at end A,

$$\text{Stiffness of the beam AB} = k = \frac{M}{\theta_A}.$$

④ Distribution factor.

When a moment is applied to a rigid joint, where a number of members are meeting, the applied moment is shared by the members meeting at that joint. The ratio of the moment shared by a member to the applied moment at the joint is called the distribution factor of that member. Thus, if M_{OA} is the moment shared by member OA when moment M is applied at joint O, then the distribution factor for member OA is.

$$d_{OA} = \frac{M_{OA}}{M}.$$



Expression for Carry over factor and stiffness.

Consider beam AB of span L shown in figure 1, In this, moment M is applied at end A, where rotation is permitted, while the end B is fixed. Let M' be the moment developed at B and θ_A be the rotation at A. As defined earlier,

$$\text{Carry over factor} = \frac{M'}{M}$$

$$\text{Stiffness of beam AB} = K = \frac{M}{\theta_A}$$

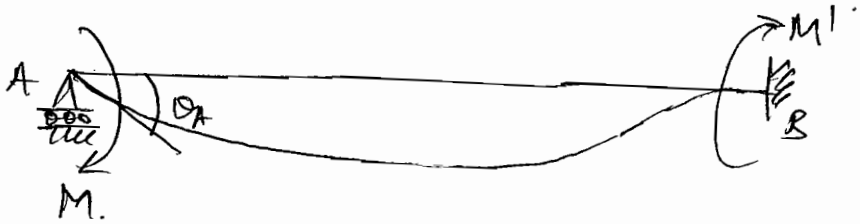


Fig 1.

To find M' & θ_A, Consistent deformation method may be used. Basic determinate structure selected is a simply supported beam as shown in fig ②. Let θ_{A1} & θ_{B1} be rotation, at ends A & B respectively. To determine these rotations Conjugate beam method may be used. Fig ③ shows such a beam with (M/EI)

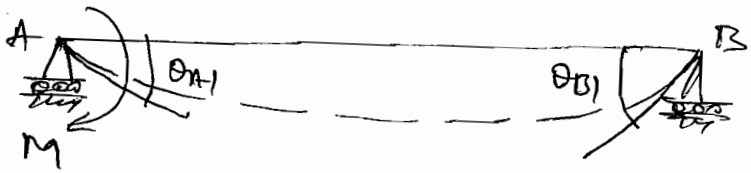


Fig 2. Basic determinate beam Subject to M.

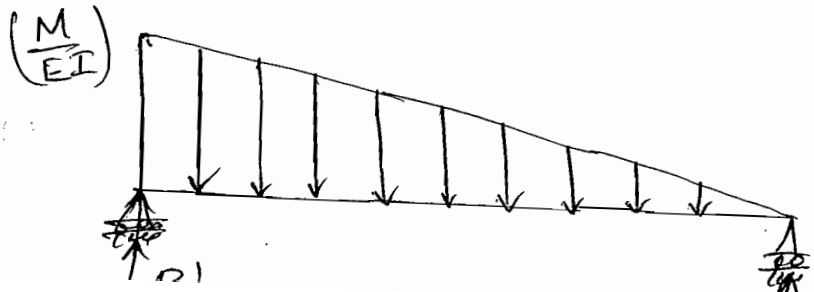


Fig 3 – Conjugate Beam

$$\theta_{A1} = R_A' = \frac{1}{3}\left(\frac{1}{2} \times L \times \frac{M}{EI}\right) = \frac{ML}{3EI} \quad \text{—— (1)}$$

$$\theta_{B1} = R_B' = \frac{1}{3}\left(\frac{1}{2} \times L \times \frac{M}{EI}\right) = \frac{ML}{6EI} \quad \text{—— (2)}$$

Now, Consider basic determinate structure subject to moment M' at B as shown in fig (4). Let θ_{A2} and θ_{B2} be the rotations at the end A and end B respectively. Conjugate beam with load diagram (M'/EI) for this case is shown in fig (5).

Fig 4 — Basic determinate beam subject to M'

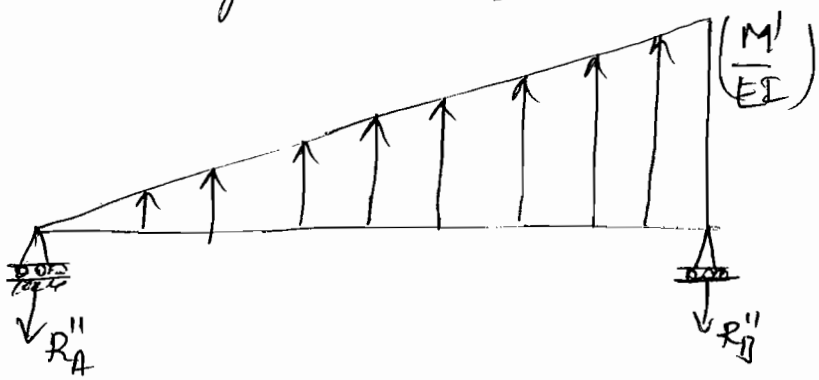


Fig 5. — Conjugate Beam.

$$\theta_{A2} = R_A'' = \frac{1}{3}\left(\frac{1}{2} \times L \times \frac{M'}{EI}\right) = \frac{M'L}{6EI} \quad \text{—— (3)}$$

$$\theta_{B2} = R_B'' = \frac{2}{3}\left(\frac{1}{2} \times L \times \frac{M'}{EI}\right) = \frac{M'L}{3EI} \quad \text{—— (4)}$$

Case 1.

Carry over factor.

$$\theta_{B1} = \theta_{B2}$$

$$\frac{ML}{6EI} = \frac{M'L}{3EI}$$

$$M' = M/2$$

$$\therefore \text{Carry Over factor} = \frac{M'}{M} = \frac{M/2}{M} = 1/2$$

$$\theta_A = \theta_{A1} - \theta_{A2}.$$

$$= \frac{ML}{3EI} - \frac{M'L}{6EI}. \qquad (M' = M/2)$$

$$= \frac{ML}{3EI} - \frac{ML}{12EI}.$$

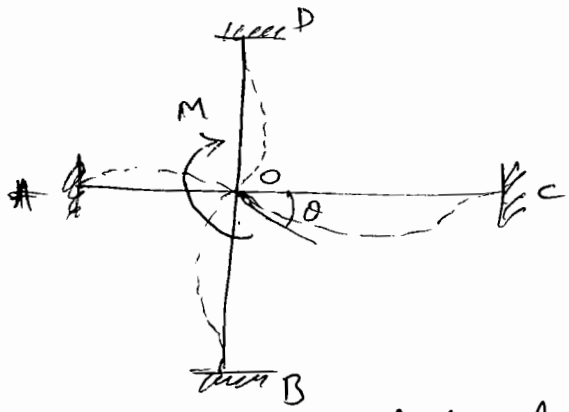
$$\cancel{= \frac{ML}{3EI}\left(1 - \frac{1}{4}\right)}$$

$$= \frac{ML}{12EI}(4-1)$$

$$\theta_A = \frac{ML}{4EI}.$$

Stiffness, $k = \frac{M}{\theta_A} = \frac{M}{ML/4EI} = \frac{4EI}{L}.$

Expression for Distribution factor.



Consider the rigid jointed plane shown in fig, in which there are four members OA, OB, OC & OD meeting at joint O. Let M be the moment applied at joint O, Since joint O is rigid, all the members rotate by the same angle 'θ'. Let M_1, M_2, M_3 & M_4 be the moments shared by members OA, OB, OC & OD respectively. Then

$$M_1 + M_2 + M_3 + M_4 = M$$

Let, k_1, k_2, k_3, & k_4 be stiffnesses and L_1, L_2, L_3 & L_4 be the lengths of members OA, OB, OC & OD respectively.

$$\qquad\qquad k_1 \qquad k_2 \qquad k_3 \qquad k_4$$

$$= \frac{M_1 + M_2 + M_3 + M_4}{k_1 + k_2 + k_3 + k_4}$$

$$\theta = \frac{M}{\sum_{i=1}^{4} k_i}$$

$$M_i = k_i \theta$$

$$M_i = k_i \left(\frac{M}{\sum_{i=1}^{4} k_i} \right)$$

Thus, a moment which is applied at a joint is shared by members meeting at the joint in proportion to their stiffnesses.

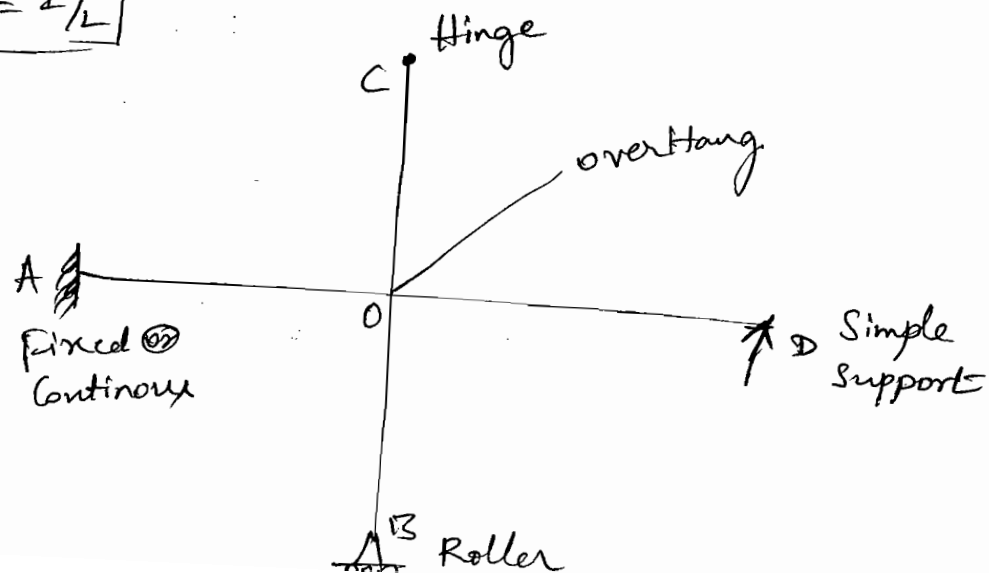
$$\therefore \text{ Distribution factor} = \frac{M_i}{M} = \frac{k_i}{\sum_{i=1}^{4} k_i}$$

Thus, distribution factor for a member is $k_i / \Sigma k$, where summation is over various members meeting at the joint. Σk is called joint stiffness.

Relative Stiffness. (k)

The ratio of M.I. to span is called relative stiffness

$$\boxed{k = I/L}$$



(i) $K = \frac{I}{l}$ → If ~~far~~ end is fixed / Continouse.

(ii) $K = \frac{3}{4}\frac{I}{l}$ → If far end is Hinged / Roller / Simple Support.

(iii) $K = 0$ → For Overhanging portion.

Ex:- Continouse Support.

(i) Wrt 'B' → 'A' – Fixed, 'C' – Continouse } $\boxed{K = I/l}$

(ii) Wrt 'C' → 'B' – Continouse – $K = I/L$

'D' – Not Continouse (S.S) – $K = 3/4 \cdot I/l$

(iii) Wrt 'D' → 'C' – Continouse – $K = I/l$

'E' – Overhanging – $K = 0$.

Steps Involved in M.D method.

① Assuming all ends are fixed, find the FEM's developed.

② Calculate distribution factors for all members meeting at a joint.

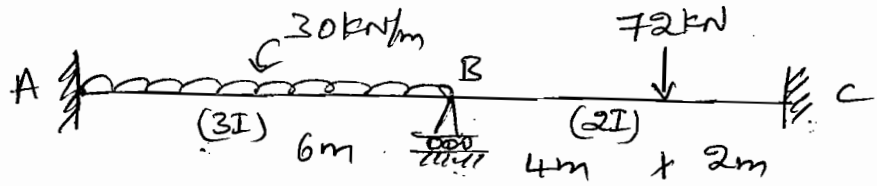
③ Balance a joint by distributing balancing moment to various members meeting at the joint proportional to their distribution factors. Do similar exercise for all joints.

④ Carry over half the distributed moment to the far ends of the members. This upsets the balance of the joint.

⑤ Repeat steps 3 and 4 till distributed moments are negligible.

⑥ Sum up all the moments at a particular end of the member to get final moment.

1) Analyze the CB shown in figure. Draw BMD & EC.



Sol:-

① FEM.

$$M_{FAB} = \frac{-wl^2}{12} = \frac{-30\times6^2}{12} = -90\,kN\text{-}m$$

$$M_{FBA} = \frac{wl^2}{12} = \frac{30\times6}{12} = 90\,kN\text{-}m$$

$$M_{FBC} = \frac{-Wab^2}{l^2} = \frac{-72\times4\times2^2}{6^2} = -32\,kN\text{-}m$$

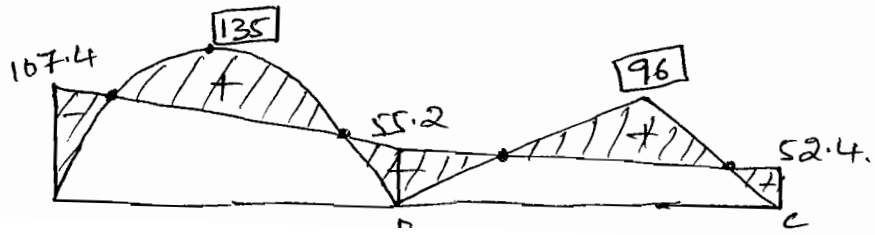
$$M_{FCB} = \frac{Wa^2b}{l^2} = \frac{+72\times4^2\times2}{6^2} = 64\,kN\text{-}m$$

② Distribution Factors. (D.F)

Joint	Members.	K	ΣK	D.F. = K/ΣK
B	BA	$I/L = \frac{3I}{6} = 0.5I$	0.833 I	$\frac{0.5}{0.833} = 0.6$
	BC	$I/L = \frac{2I}{6} = 0.33I$		$\frac{0.33}{0.833} = 0.4$

③ Distribution Table.

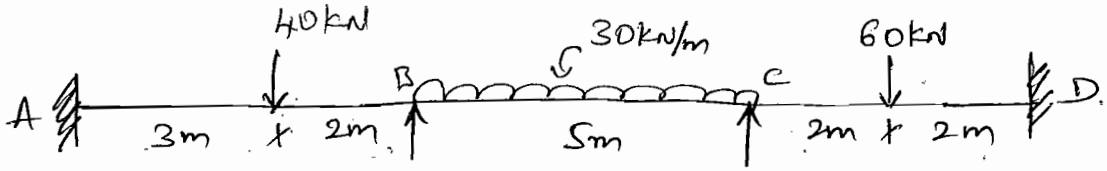
A	B		C	Members.
	0.6	0.4		DF.
−90	90	−32	64	FEM.
	−34.8	−23.2		Balancing
−17.4			−11.6.	C.O.M.
−107.4.	55.2	−55.2	52.4.	FM.



$$BM)_{AB} = \frac{WL^2}{8} = \frac{30\times6^2}{8} = 135\,kN\text{-}m$$

$$BM)_{BC} = \frac{Wab}{l} = \frac{72\times4\times2}{6} = 96\,kN\text{-}m$$

(2) Analyze the C.B shown in figure. Draw B.M & S.F



Sol:

① FEM.

$$M_{FAB} = \frac{-Wab^2}{l^2} = \frac{-40\times3\times2^2}{5^2} = -19.2\,kN\text{-}m$$

$$M_{FBA} = \frac{+Wa^2b}{l^2} = \frac{40\times3^2\times2}{5^2} = 28.8\,kN\text{-}m$$

$$M_{FBC} = \frac{-Wl^2}{12} = \frac{-30\times5^2}{12} = -62.5\,kN\text{-}m$$

$$M_{FCB} = \frac{Wl^2}{12} = 62.5\,kN\text{-}m$$

$$M_{FCD} = \frac{-Wl}{8} = \frac{-60\times4}{8} = -30\,kN\text{-}m$$

$$M_{FDC} = \frac{+Wl}{8} = 30\,kN\text{-}m$$

② Distribution Factors.

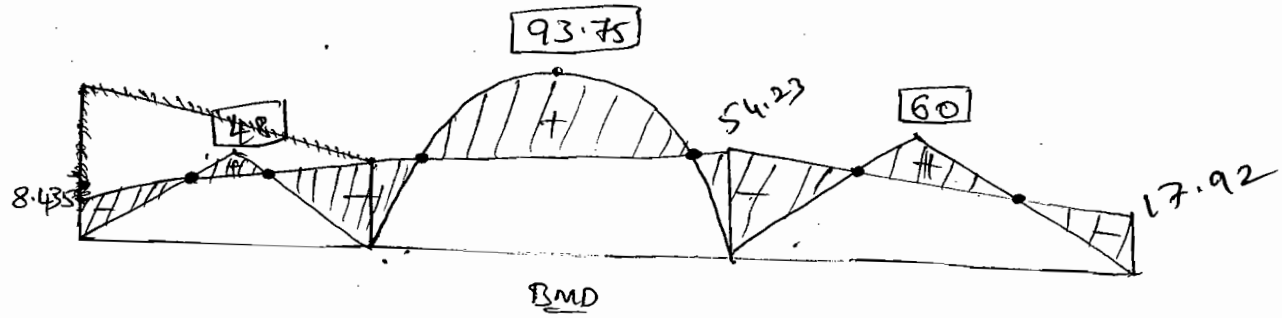
Joints	Members	K	ΣK	$\left(\frac{K}{\Sigma K}\right)$ = D.F.
B	BA	$I/L = \frac{I}{5} = 0.2I$	0.4I	0.5
	BC	$I/L = I/5 = 0.2I$		0.5
C	CB	$I/L = I/5 = 0.2I$	0.45I	0.44
	CD	$I/L = I/4 = 0.25I$		0.56

A	B		C		D	Members
	0.5	0.5	0.44	0.56		D.F.
−19.2	28.8	−62.5	62.5	−30	30	FEM
	16.85	16.85	−14.3	−18.2		Bal
8.425		−7.15	8.425		−9.1	C.O.M
	3.57	3.57	−3.70	−4.71		Bal
1.78		−1.85	1.78		−2.35	COM
	0.925	0.925	−0.78	−1.00		Bal
0.46		−0.39	0.46		−0.5	COM
	0.19	0.19	−0.20	−0.26		Bal
0.10		−0.1	0.10		−0.13	COM
	0.05	0.05	−0.044	−0.056		Bal
−8.435	50.385	−50.40	54.24	−54.23	17.92	Final Moment

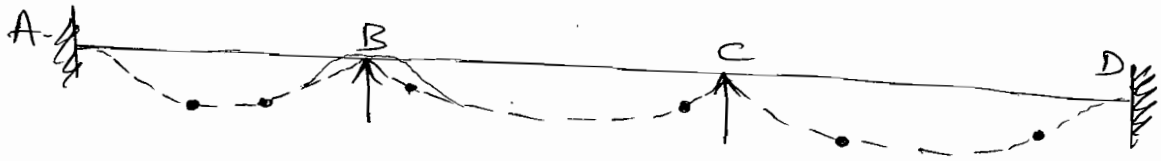
$$BM)_{AB} = \frac{Wab}{l} = \frac{40\times3\times2}{5} = 48\ kN\text{-}m$$

$$BM)_{BC} = \frac{Wl^2}{8} = \frac{30\times5^2}{8} = 93.75\ kN\text{-}m$$

$$BM)_{CD} = \frac{Wl}{4} = \frac{60\times4}{4} = 60\ kN\text{-}m$$

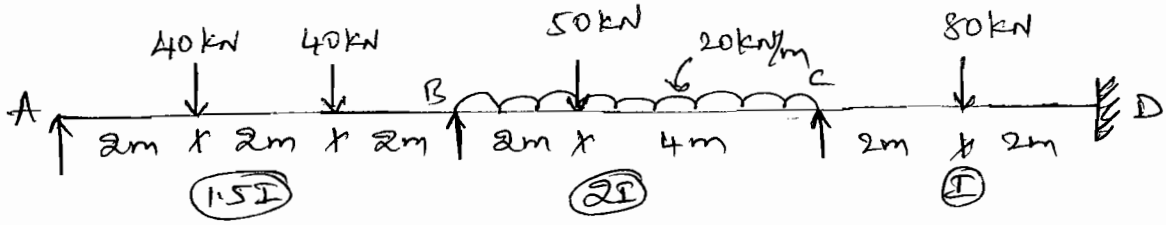


BMD



EC

3) Analyze the C.B system in figure, by the [illegible]



Sol:

① FEM.

$$M_{FAB} = -\left(\frac{Wab^2}{l^2} + \frac{Wab^2}{l^2}\right) = -\left(\frac{40\times2\times4^2}{6^2} + \frac{40\times4\times2^2}{6^2}\right) = -53.33\ kN\text{-}m$$

$$M_{FBA} = \left(\frac{Wa^2b}{l^2} + \frac{Wa^2b}{l^2}\right) = \left(\frac{40\times2^2\times4}{6^2} + \frac{40\times4\times2^2}{6^2}\right) = 53.33\ kN\text{-}m$$

$$M_{FBC} = \frac{-Wl^2}{12} - \frac{Wab^2}{l^2} = -\frac{20\times6^2}{12} - \frac{50\times2\times4^2}{6^2} = -104.44\ kN\text{-}m$$

$$M_{FCB} = \frac{+20\times6^2}{12} + \frac{50\times2^2\times4}{6^2} = 82.22\ kN\text{-}m$$

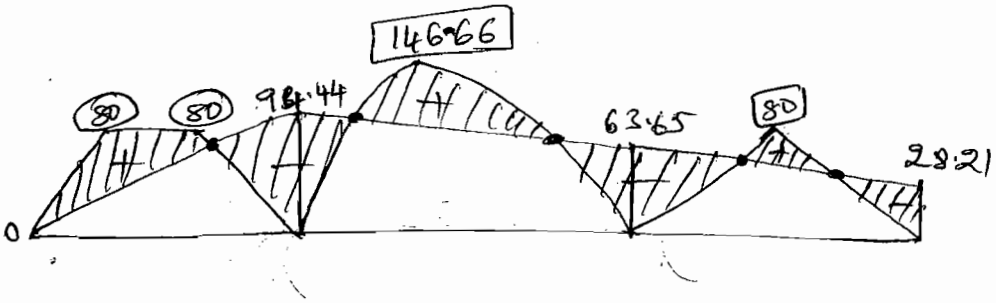
$$M_{FCD} = \frac{-Wl}{8} = -40\ kN\text{-}m$$

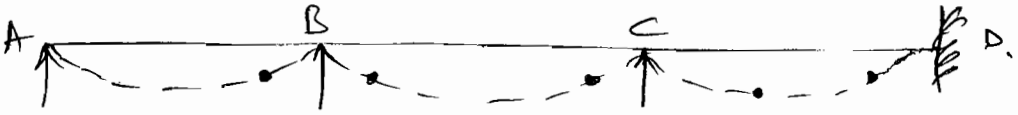
$$M_{FDC} = \frac{Wl}{8} = 40\ kN\text{-}m$$

② D.F.

Joint	Members.	k	Σk	$\left(\frac{k}{\Sigma k}\right)$ = D.F.
B	BA	$3/4(I/L) = 3/4(1.5I/6) = 0.1875I$	0.5205I	0.36
	BC	$I/L = 2I/6 = 0.333I$		0.64
C	CB	$I/L = 2I/6 = 0.333I$	0.583I	0.57
	CD	$I/L = I/4 = 0.25I$		0.43.

A (S.S/Roller Hinge)		B		C		D	Members.
	0.36	0.64		0.57	0.43		D.F.
-53.33	53.33	-104.44		82.22	-40	40	FEM.
53.33 →	26.66.						Release (A). COM.
0	80	-104.44		82.22	-40	40	Initial Values
	8.79	15.64		-24.06	-18.15		Bal.
0		-12.03		7.82		-9.07	COM.
	4.33	7.69		-4.45	-3.36		Bal.
0		-2.22		3.84		-1.68	Com.
	0.79	1.42		-2.18	-1.65		Bal
0		-1.09		0.71		-0.82	COM.
	0.39	0.69		-0.40	-0.31		Bal
0		-0.20		+0.34		-0.15	COM
	0.07	0.128		-0.19	-0.15		Bal
0		-0.1		0.064		-0.07	COM.
	0.036	0.064		-0.037	-0.027		Bal
0	94.44	-94.44		63.674	-63.647	28.21	Final Moments.





$BM)_{AB} = \frac{40 \times 6}{3} = 80 \text{ kNm}$

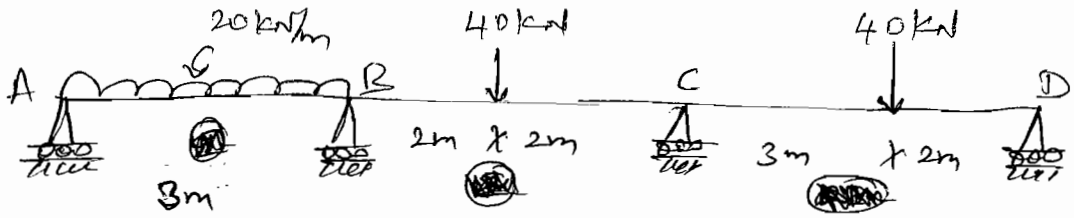
$BM)_{BC} \Rightarrow R_B = 93.33$

$R_C = 76.66$

$\therefore M_E = 93.33 \times 2 - \frac{20 \times 2^2}{2}$

$= 146.66 \text{ kNm}$

$BM)_{CD} = \frac{Wl}{4} = \frac{80 \times 4}{4} = 80 \text{ kNm}$



Soln

① FEM.

$$M_{FAD} = -\frac{Wl^2}{12} = -\frac{20\times 3^2}{12} = -15\ kN\text{-}m$$

$$M_{FBA} = \frac{Wl^2}{12} = 15\ kN\text{-}m$$

$$M_{FBC} = -\frac{Wl}{8} = \frac{-40\times 4}{8} = -20\ kN\text{-}m$$

$$M_{FCB} = \frac{Wl}{8} = 20\ kN\text{-}m$$

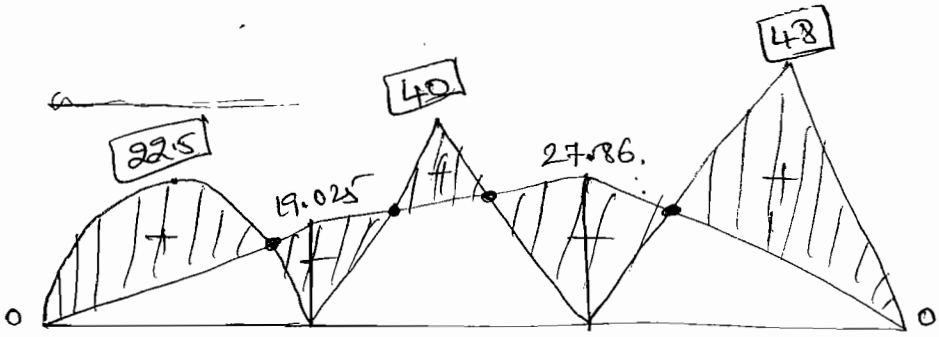
$$M_{FCD} = \frac{-Wab^2}{l^2} = \frac{-40\times 3\times 2^2}{5^2} = -19.2\ kN\text{-}m$$

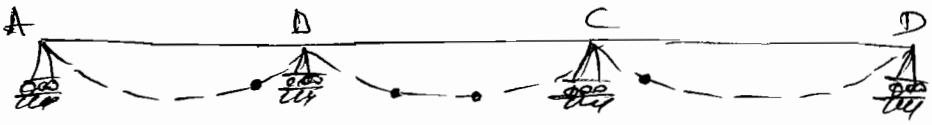
$$M_{FDC} = \frac{Wa^2b}{l^2} = \frac{+40\times 3^2\times 2}{5^2} = 28.8\ kN\text{-}m$$

② D.F.

Joint	Members	K	ΣK	D.F
B	BA	$\frac{3}{4}(I/3) = 0.25I$	0.5I	0.5
	BC	$I/4 = 0.25I$		0.5
C	CB	$I/4 = 0.25I$	0.4I	0.625
	CD	$\frac{3}{4}(I/5) = 0.15I$		0.375

A	B		C		D	Members.
	0.5	0.5	0.625	0.375		DF
	15	−20	20	−19.2	28.8	FEM
−15					−28.8	Release (A) (D)
15	7.5			−14.4		COM
0	22.5	−20	20	−33.6	0	Initial values
	−1.25	−1.25	8.5	5.1		Bal.
		4.25	−0.625			COM
	−2.125	−2.125	0.39	0.234		Bal
		0.2	−1.06			COM
	−0.1	−0.1	0.662	0.397		Bal
0	19.025	−19.025	27.867	−27.869	0	Final Moments.





$$BM)_{AB} = \frac{wl^2}{8} = \frac{20 \times 3^2}{8}$$

$$= 22.5 \text{ kN-m}$$

$$BM)_{BC} = \frac{wl}{4} = \frac{40 \times 4}{4}$$

$$= 40 \text{ kN-m}$$

$$BM)_{CD} = \frac{wab}{l} = \frac{40 \times 3 \times 2}{5}$$

$$= 48 \text{ kN-m}$$

(a) Near end Rotation $= m = \frac{4EI\theta}{l}$

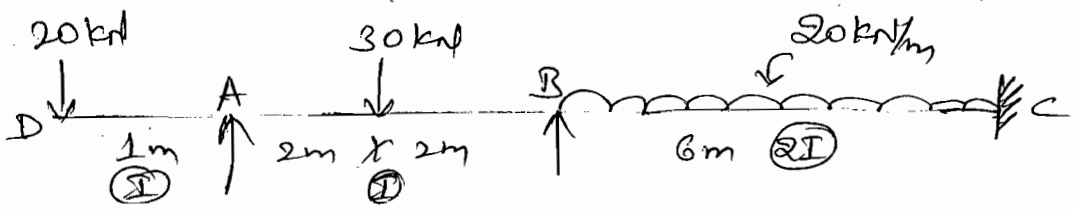
(b) Far end Rotation $= m = \frac{2EI\theta}{l}$

(c) Due to Sinking $= m = -\frac{6EI\Delta}{l^2}$

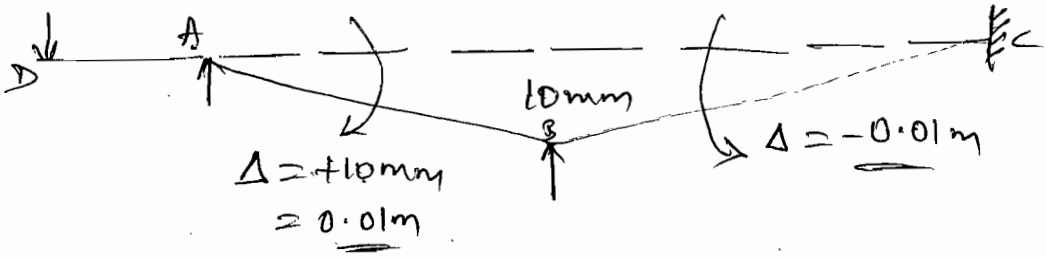
The above additional moments are added to FEM's.

Where, θ = Rotation, Δ = Sinking.

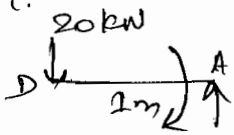
(5) Analyze the CB shown in figure, The support 'B' sinks by 10mm. Take EI = 4000 kN-m². Draw BMD & EC.



Sol:-



(1) FEM.



$M_{AD} = 20 \times 1 = 20$ kN-m

$M_{FAB} = -\frac{Wl}{8} - \frac{6EI\Delta}{l^2} = -\frac{30\times4}{8} - \frac{6(1\times4000)(0.01)}{4^2} = -30$ kN-m

$M_{FDA} = \frac{Wl}{8} - \frac{6EI\Delta}{l^2} = 15 - 15 = 0$

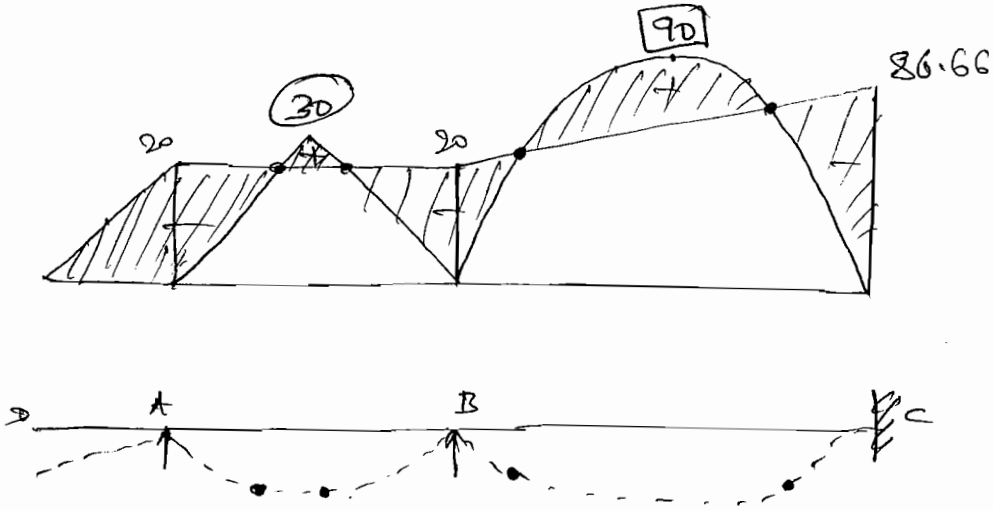
$M_{FBC} = -\frac{Wl^2}{12} - \frac{6EI\Delta}{l^2} = -\frac{20\times6^2}{12} - \frac{6(2\times4000)(-0.01)}{6^2} = -46.67$ kN-m

$M_{FCB} = \frac{20\times6^2}{} \quad \frac{6(2\times4000)(-0.01)}{} = 73.33$ kN-m

Joint	Member	K	ΣK	DF.
A	AD	0	0.25I	0
	AB	$I/4 = 0.25I$		1
B	BA	$\frac{3}{4}(I/4) = 0.1875I$	0.5205 I	0.36
	BC	$\frac{2I}{6} = 0.333I$.		0.64

(B) M.D. Table

D	A		B		C	Members
	0	1	0.36	0.64		D.F.
	20	-30	0	-46.67	73.33	FEM.
	0	10	16.80	29.86		Bal.
			5		14.93	COM.
		0	-1.8	-3.2		Bal.
					-1.6.	COM.
	20	-20	20	-20	86.66.	Final Moments.



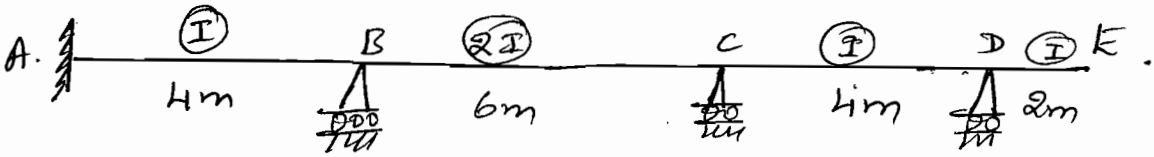
$BM)_{AB} = \frac{Wl}{4} = \frac{30 \times 4}{4}$

$= 30$ kN-m

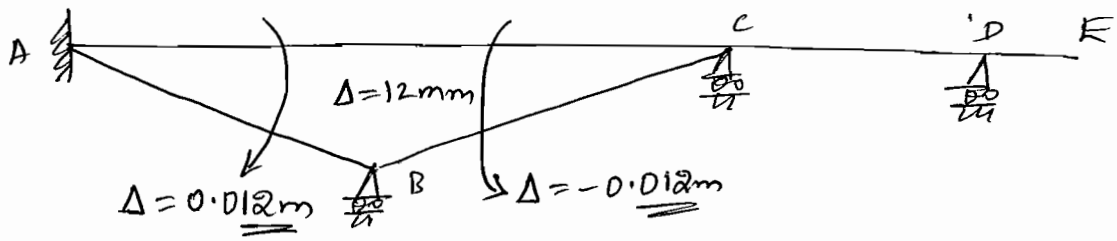
$BM)_{BC} = \frac{Wl^2}{8} = \frac{20 \times 6^2}{8}$

$= 90$ kN-m

(6) Analyse the C.B shown in figure, if support B sinks by 12mm. Given $E = 200 kN/mm^2$ & $I = 20 \times 10^6 mm^4$.



Sol:-



$$EI = 200 \times 20 \times 10^6 \ kN\text{-}mm^2.$$

$$= \frac{4000 \times \cancel{10^6}}{\cancel{10^6}} \ kN\text{-}m^2$$

(1) FEM.

$$M_{FAB} = \frac{-6EI\Delta}{l^2} = \frac{-6 \times 4000 \times 0.012}{4^2} = -18 kN\text{-}m$$

$$M_{FBA} = \frac{-6EI\Delta}{l^2} = -18 kN\text{-}m$$

$$M_{FBC} = \frac{-6EI\Delta}{l^2} = \frac{-6 \times 2 \times 4000 \times (-0.012)}{6^2} = 16 kN\text{-}m$$

$$M_{FCB} = \frac{-6EI\Delta}{l^2} = \frac{-6 \times 2 \times 4000 (-0.012)}{6^2} = 16 kN\text{-}m$$

$M_{FCD} = 0$ $\quad M_{FDC} = 0$

No FEM for Overhanging span DE.

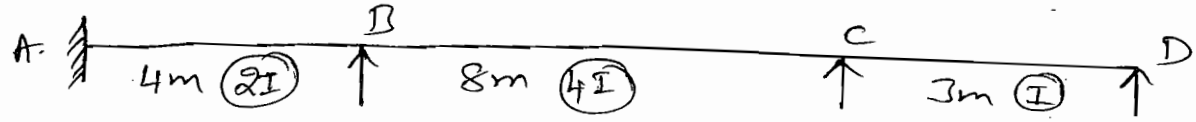
② D.F

Joint	Members	K	ΣK	$DF = \frac{k}{\Sigma k}$
B	BA	$I/L = I/4 = 0.25I$	$0.58I$	0.43
	BC	$I/L = 2I/6 = 0.33I$		0.57
C	CB	$I/L = 2I/6 = 0.33I$	$0.5205I$	0.64
	CD	$\frac{3}{4}\left(\frac{I}{L}\right) = 3/4\,(I/4) = 0.1875I$		0.36
D	DC	$I/L = I/4 = 0.25I$	$0.25I$	1
	DE	0.		0

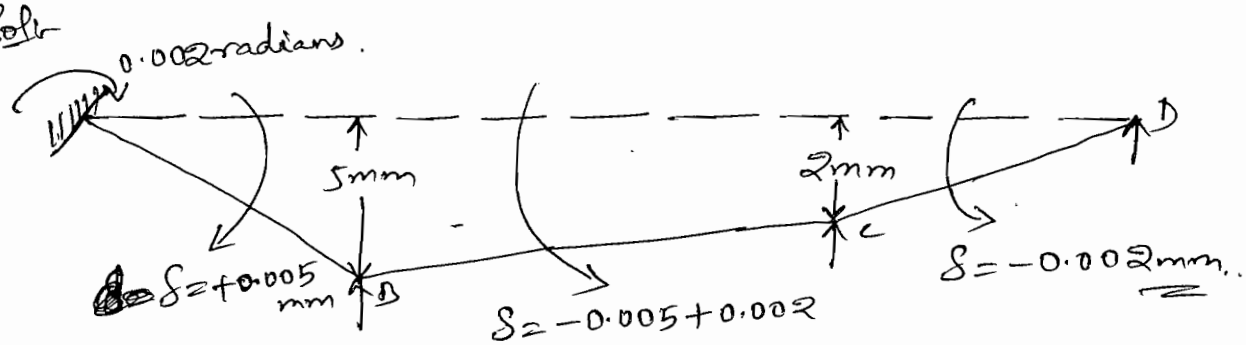
③ M.D.T.

A	B		C		D		E	Members
	0.43	0.57	0.64	0.36	1	0.		DF
−18	−18	16	16	0	0	0	0.	FEM.
	0.86	1.14	−10.24	−5.76				Bal M.
0.43		5.12	0.57					COM.
	−2.20	−2.91	−0.36	−0.21				Bal M
−1.10		−0.18	−1.455					COM.
	0.077	0.102	0.93	0.52				Bal M.
0.0385		0.465	0.051					COM.
	−0.199	−0.265	−0.032	−0.018				Bal M.
−18.63.	−19.462	19.47	5.464	−5.468	0	0	0.	Final Moments.

(7) Figure shows a Continous Beam ABCD, analyse the beam, if the end 'A' rotates by 0.002 radians in clockwise order & support 'B' sinks by 5mm & 'C' by 2mm. Take $EI = 18000\ kN\text{-}m^2$.



Soln



(1) FEM.

$$M_{FAB} = 0 + \frac{4EI\theta}{L} - \frac{6EI\delta}{l^2} = \frac{4\times2\times18000\times0.002}{4} - \frac{6\times2\times18000\times0.005}{4^2}$$

$$= 4.5\ kN\text{-}m$$

$$M_{FBA} = \frac{2EI\theta}{l} - \frac{6EI\delta}{l^2} = \frac{2\times2\times18000\times0.002}{4} - \frac{6\times2\times18000\times0.005}{4^2}$$

$$= -31.5\ kN\text{-}m$$

$$M_{FBC} = 0 - \frac{6EI\delta}{l^2} = -\frac{6\times4\times18000\times(-0.003)}{8^2} = 20.25\ kN\text{-}m$$

$$M_{FCB} = 0 - \frac{6EI\delta}{l^2} = -\frac{6\times4\times18000\times(-0.003)}{8^2} = 20.25\ kN\text{-}m$$

$$M_{FCD} = 0 - \frac{6EI\delta}{l^2} = -\frac{6\times18000\times(-0.002)}{3^2} = 24\ kN\text{-}m$$

$$M_{FDC} = 0 - \frac{6EI\delta}{l^2} = -\frac{6\times18000\times(-0.002)}{3^2} = 24\ kN\text{-}m$$

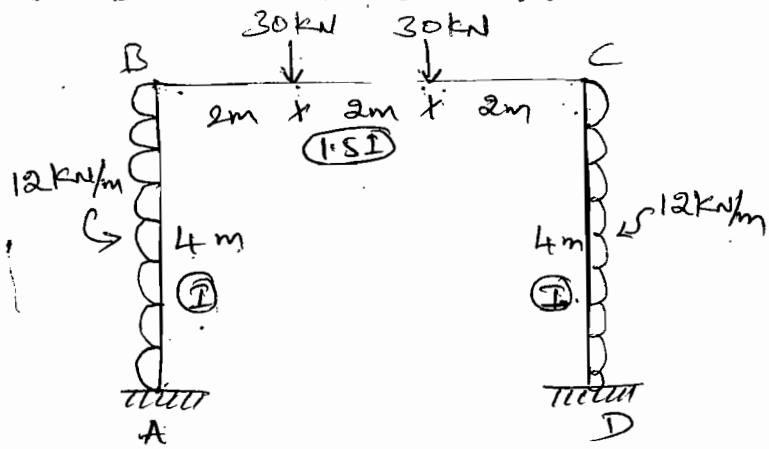
② D.F.

Joints	Members	K	ΣK	DF = K/ΣK
B	BA	$I/L = \frac{2I}{4} = 0.5I$	I	0.5
	BC	$I/L = \frac{4I}{8} = 0.5I$.		0.5
C	CB	$I/L = \frac{4I}{8} = 0.5I$.	0.75I	0.67
	CD	$\frac{3}{4} \times \frac{I}{L} = \frac{3}{4} \times \frac{I}{3} = 0.25I$		0.33.

③. M.D.T.

A	B		C		D	Members.
	0.5	0.5	0.67	0.33		D.F.
4.5	−31.5	20.25	20.25	24	24	FEM.
				−12. ←	−24	Release @ D.
4.5	−31.5	20.25	20.25	12	0	Initial Moments
	5.625	5.625	−21.60	−10.64		Bal M.
2.812		−10.8	2.812.			COM.
	5.4	5.4	−1.88	−0.92		Bal. M
2.7		−0.94	2.7			COM.
	0.47	0.47	−1.809	−0.891		Bal. M.
0.235		−0.904	0.235			COM.
	0.452	0.452	−0.157	−0.077		Bal. M
0.226		−0.078	0.226			COM.
	0.039	0.039	−0.151	−0.074		Bal. M.
10.473	−19.514	19.514	0.626	−0.602	0	Final Moments.

(6) Analyze the frame shown in figure. Draw B.M.D



Sol:

① FEM.

$$M_{FAB} = -\frac{Wl^2}{12} = -\frac{12\times4^2}{12} = -16\ kNm$$

$$M_{FBA} = \frac{Wl^2}{12} = \frac{12\times4^2}{12} = +16\ kNm$$

$$M_{FBC} = -\left(\frac{Wab^2}{l^2} + \frac{Wab^2}{l^2}\right) = -\left(\frac{30\times2\times4^2}{6^2} + \frac{30\times4\times2^2}{6^2}\right) = -40\ kNm$$

$$M_{FCB} = +\left(\frac{Wa^2b}{l^2} + \frac{Wa^2b}{l^2}\right) = \left(\frac{30\times2^2\times4}{6^2} + \frac{30\times4^2\times2}{6^2}\right) = 40\ kNm$$

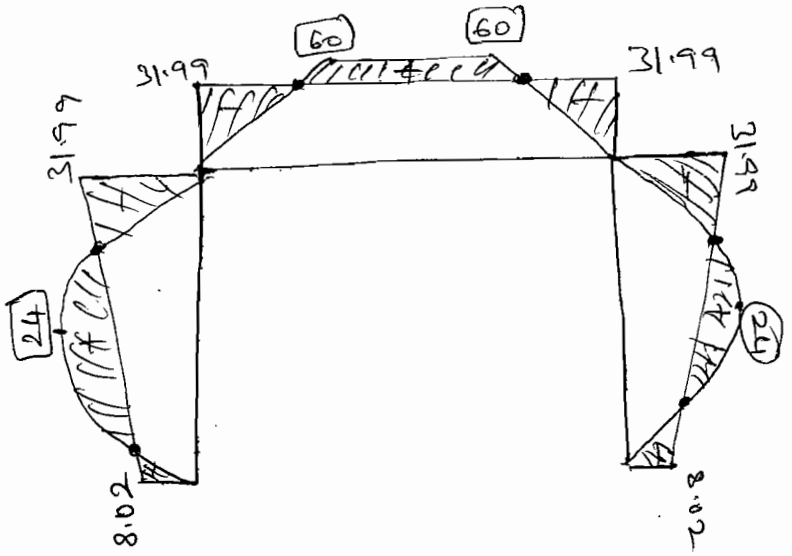
$$M_{FCD} = -\frac{Wl^2}{12} = -16\ kNm$$

$$M_{FDC} = \frac{Wl^2}{12} = 16\ kNm$$

② D.F.

Joint	Members	K	ΣK	$DF = \frac{K}{\Sigma K}$
B	BA	$I/L = I/4 = 0.25I$	0.5I	0.5
	BC	$I/L = 1.5I/6 = 0.25I$		0.5
C	CB	$I/L = \frac{1.5I}{6} = 0.25I$	0.5I	0.5
	CD	$I/L = I/4 = 0.25I$		0.5

A	B		C		D	Members.
	0.5	0.5	0.5	0.5		DF
-16	16	-40	40	-16	16	FEM.
	12	12	-12	-12		Bal
6		-6	6		-6	COM.
	3	3	-3	-3		Bal
1.5		-1.5	1.5		-1.5	COM.
	0.75	0.75	-0.75	-0.75		Bal
0.38		-0.38	0.38		-0.38	COM
	0.19	0.19	-0.19	-0.19		Bal
0.1		-0.1	0.1		-0.1	COM.
	+0.05	+0.05	-0.05	-0.05		Bal.
-8.02	31.99	-31.99	31.99	-31.99	+8.02	Final Moments.



$$BM)_{AB} = \frac{Wl^2}{8} = BM)_{CD}$$

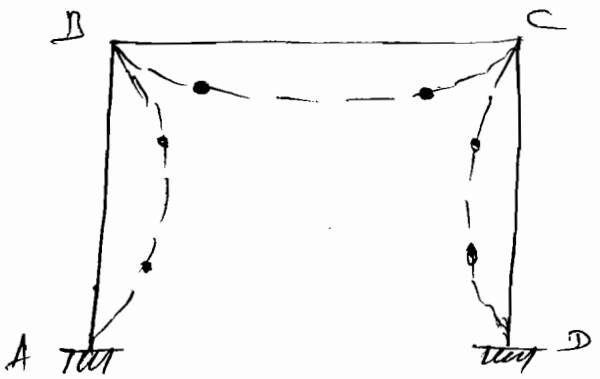
$$= \frac{12 \times 4^2}{8}$$

$$= 24 \text{ kN-m}$$

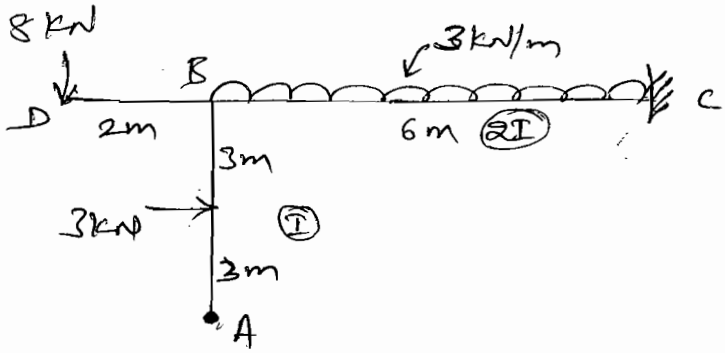
$$BM)_{BC} = Wl/3$$

$$= \frac{30 \times 6}{3}$$

$$= 60 \text{ kN-m}$$



(4) Analyse the frame shown in figure by slope deflection method.



Sol:

(1) FEM.

$$M_{FAD} = \frac{-Wl}{8} = \frac{3\times6}{8} = -2.25\ kN\text{-}m$$

$$M_{FBA} = \frac{Wl}{8} = 2.25\ kN\text{-}m$$

$$M_{FBC} = \frac{-Wl^2}{12} = \frac{3\times6^2}{12} = -9\ kN\text{-}m$$

$$M_{FCB} = \frac{Wl^2}{12} = 9\ kN\text{-}m$$

$$M_{BD} = +8\times2 = 16\ kN\text{-}m$$

(2) D.F.

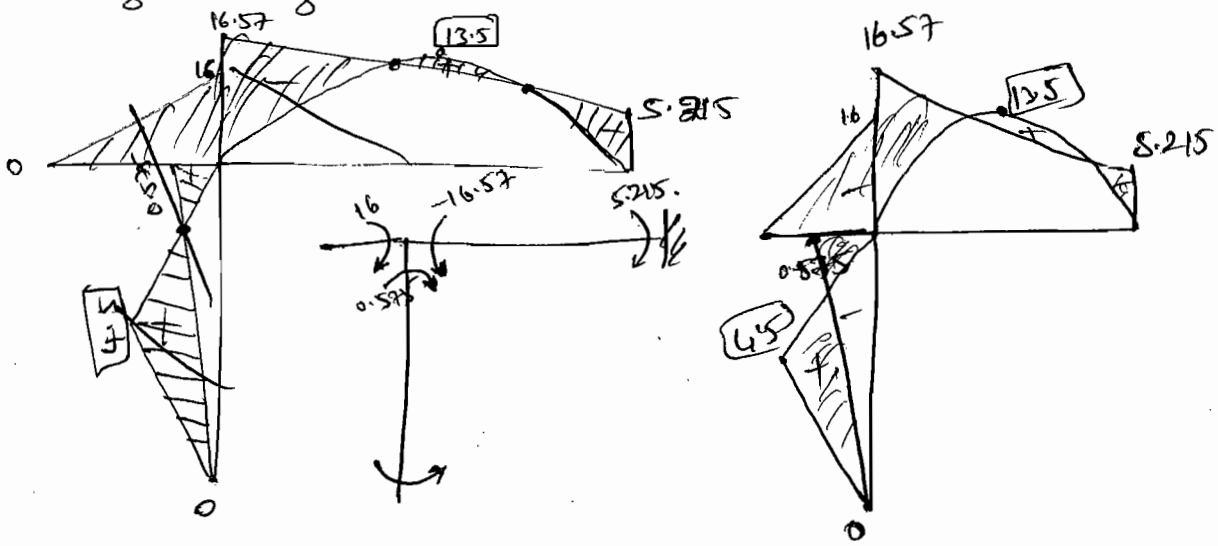
Joint	Members	K	ΣK	$\left(\frac{K}{\Sigma K}\right)$ DF
B	BA	$\frac{3}{4}\left(\frac{I}{6}\right) = 0.125I$	0.458I	0.27
	BC	$\frac{2I}{6} = 0.333I$		0.73
	BD	0 ($\because$ overhanging)		0.

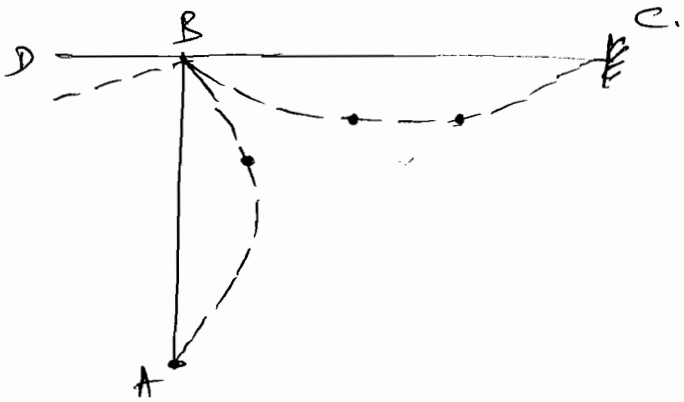
③ M.D Table

A	B		C		Members.
AB	BA	BB	BC	CB.	
	0.27	0	0.73		DF.
	2.25	16	−9	+9	FEM.
−2.25					
2.25 →	1.125				Release @ A
					COM
0	3.375	16	−9	+9	Initial Moment
	−2.80	0	−7.57 →	−3.785	Bal
					COM.
0	0.575	16	−16.57	5.215.	Final Moments

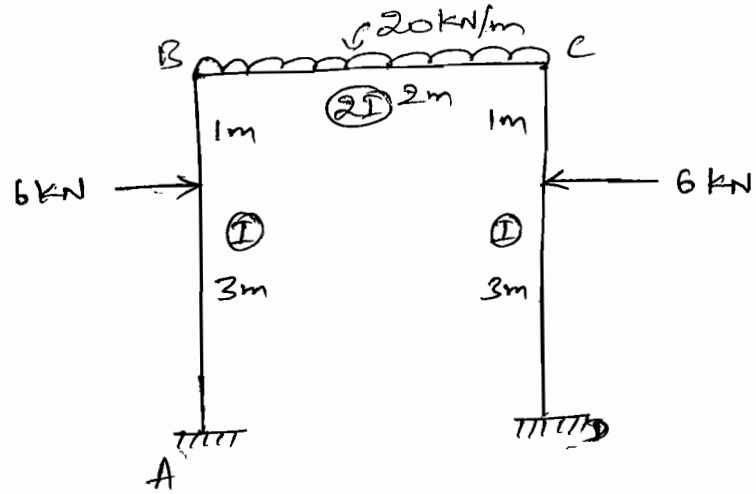
$$BM)_{AB} = \frac{Wl}{4} = \frac{3\times 6}{4} = 4.5\,kN\text{-}m$$

$$BM)_{BC} = \frac{+Wl^2}{8} = \frac{3\times 6^2}{8} = 13.5\,kN\text{-}m$$





(6) Analyse the frame shown in figure



Sol:-

(1) FEM.

$$M_{FAB} = \frac{-Wab^2}{l^2} = -\frac{6\times3\times1^2}{4^2} = -1.125 \text{ kN-m}$$

$$M_{FBA} = \frac{Wa^2b}{l^2} = \frac{6\times3^2\times1}{4^2} = 3.375 \text{ kN-m}$$

$$M_{FBC} = -\frac{Wl^2}{12} = -\frac{20\times2^2}{12} = -6.67 \text{ kN-m}$$

$$M_{FCB} = \frac{Wl^2}{12} = \frac{20\times2^2}{12} = 6.67 \text{ kN-m}$$

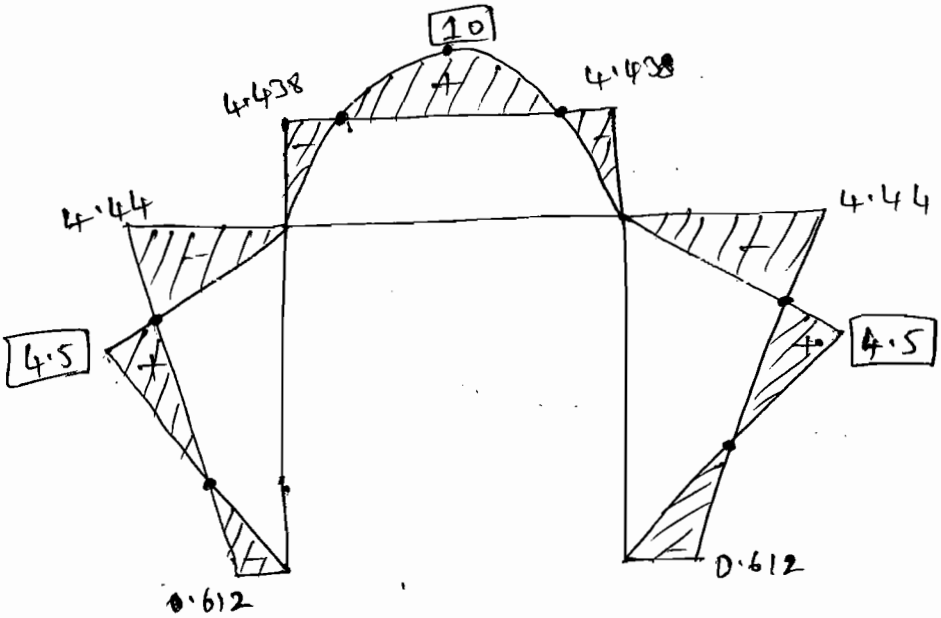
$$M_{FCD} = -\frac{Wab^2}{l^2} = -\frac{6\times1\times3^2}{4^2} = -3.375 \text{ kN-m}$$

$$M_{FDC} = -\frac{Wa^2b}{l^2} = -\frac{6\times1^2\times3}{4^2} = 1.125 \text{ kN-m}$$

(2) D.F.

Joint	Members	K	ΣK	D.F.
B	BA	$\frac{I}{L} = \frac{I}{4} = 0.25I$	1.25I	0.2
	BC	$\frac{2I}{L} = \frac{2I}{2} = I$		0.8
C	CB	$\frac{2I}{2} = I$	1.25I	0.8
	CD	$I/4 = 0.25I$		0.2

A		B		C		D	Members
	0.2	0.8		0.8	0.2		DF
−1.125	3.375	−6.67		6.67	−3.375	+1.125	FEM.
	0.66	2.64		−2.64	−0.66		Bal M.
0.33		−1.32		1.32		−0.33	COM.
	0.26	1.06		−1.06	−0.26		Bal. M
0.13		−0.53		0.53		−0.13	COM
	0.106	0.424		−0.424	−0.106		Bal M.
0.053		−0.212		0.212		−0.053	COM.
	0.042	0.17		−0.17	−0.042		Bal M
−0.612	4.44	−4.438		4.438	−4.44	0.612	Final moments



BM)$_{AB \& CD}$

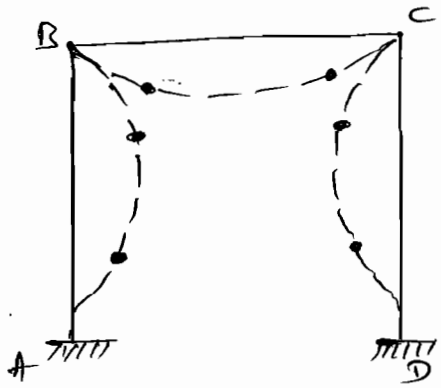
$= \frac{Wab}{l} = \frac{6 \times 1 \times 3}{4}$

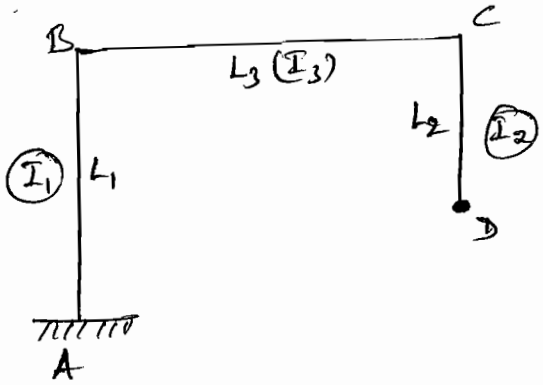
$= 4.5$ kN-m

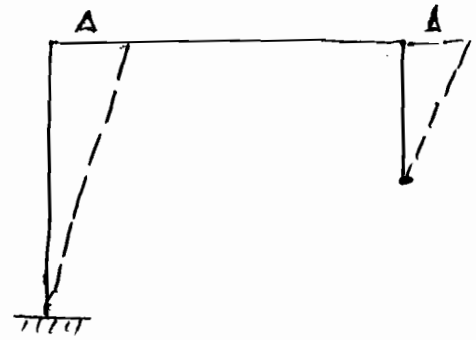
BM)$_{BC}$

$= \frac{Wl^2}{8} = \frac{20 \times 2^2}{8}$

$= 10$ kN-m







Additional moment due to sway.

(i) If far end is fixed/Continous.

$$m = -\frac{6EI\delta}{l^2}$$

(ii) If far end is Hinged/Roller/SS.

$$m = -\frac{3EI\delta}{l^2}$$

} Taken only for Vertical members.

Final Moments.

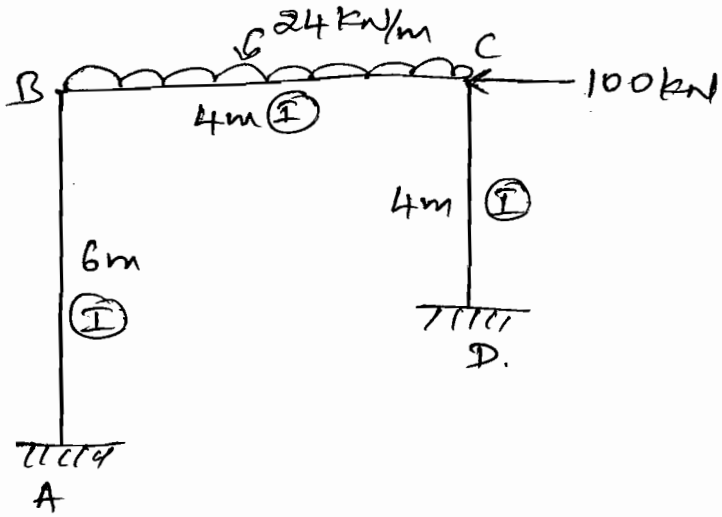
$$\boxed{M = \bar{M} + K \cdot M'}$$

$\bar{M}$ = Non-Sway Moments M' = Sway Moments.

K = Correction factor.

Problems.

1) Analyze the frame by M.D. method, Draw BMD & EC.



$M_{FAB} = M_{FBA} = 0$

$M_{FCD} = M_{FDC} = 0.$

$M_{FBC} = -\frac{wl^2}{12} = -\frac{24 \times 4^2}{12} = -32\,kN\text{-}m$

$M_{FCB} = 32\,kN\text{-}m$

② D.F.

Joint	Members	k	Σk	D.F.
B	BA	$I/6 = 0.167I$	$0.417I$	0.4
	BC	$I/4 = 0.25I$		0.6.
C	CB	$I/4 = 0.25I$	$0.5I$	0.5
	CD	$I/4 = 0.25I$		0.5

③ Non-Sway Analysis. (M)

A		B		C		D	Members
	0.4	0.6		0.5	0.5		D.F.
0	0	−32		32	0	0	FEM
	12.8	19.2		−16	−16		Bal M
6.4		−8		9.6		−8	COM
	3.2	4.8		−4.8	−4.8		Bal M
1.6		−2.4		2.4		−2.4	COM.
	0.96	1.44		−1.2	−1.2		Bal M
0.48		−0.6		0.72		−0.6	COM
	0.24	0.36		−0.36	−0.36		Bal M
0.12		−0.18		0.18		−0.18	COM
	0.072	0.108		−0.09	−0.09		Bal M
8.6	17.27	−17.27		22.45	−22.45	−11.18.	Final (NM ~~Moments~~ Values.

The above values are M

Additional moment due to sway.

$$M = \frac{-6EI\delta}{l^2}$$

$$M_{AB} = M_{BA} = -\frac{6EI\delta}{6^2} = -\frac{EI\delta}{6}.$$

$$M_{CD} = M_{DC} = -\frac{6EI\delta}{4^2} = -\frac{6EI\delta}{16}.$$

$$\frac{M_{AB}}{M_{DC}} = \frac{M_{BA}}{M_{CD}} = \frac{1/6}{6/16} = \frac{1}{6} \times \frac{16}{6} = \left(\frac{4}{9}\right)$$

Assume any suitable values, but according to the above ratio.

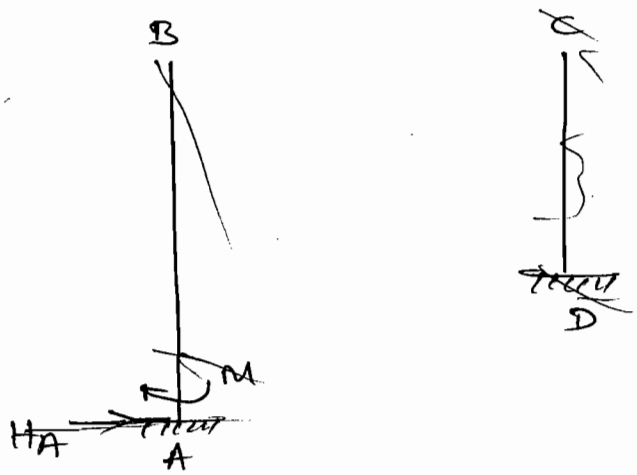
$$M_{AB} = M_{BA} = 4 \text{ kN-m}$$

$$M_{CD} = M_{DC} = 9 \text{ kN-m}.$$

A	B		C		D	Members
	0.4	0.6	0.5	0.5		DF
4	4	0	0	9	9	Mom
	-1.6	-2.4	-4.5	-4.5	-2.25	Bal M
-0.8		-2.25	-1.2			COM
	0.9	1.35	0.6	0.6	0.3	Bal M
0.45		0.3	0.675			COM
	-0.12	-0.18	-0.337	-0.337	-0.168	Bal M
-0.06		-0.168	-0.09			COM
	0.067	0.100	0.045	0.045		Bal M
3.59	3.24	-3.24	-4.81	4.81	6.88	Final values

The above values are M'.

⑤ Calculation of correction factor using Shear Condition



⑤ Final Moments.

$$M = \bar{M} + K(M')$$

$M_{AB} = 8.6 + K(3.59)$

$M_{BA} = 17.27 + (3.24)K$

$M_{BC} = -17.27 - (3.24)K$

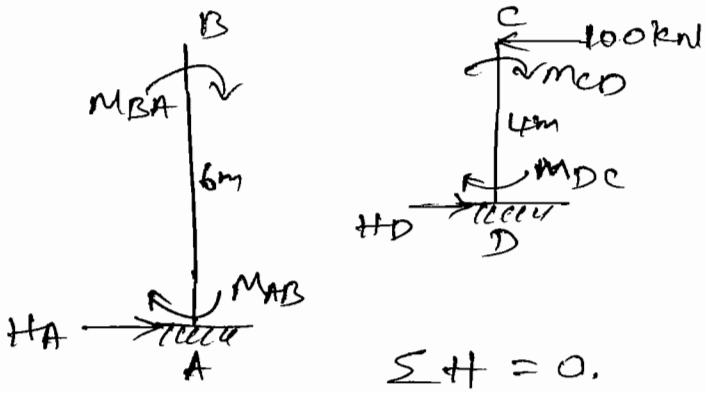
$M_{CB} = 22.45 - (4.81)K$

$M_{CD} = -22.45 + (4.81)K$

$M_{DC} = -11.18 + (6.88)K$

— (I)

⑥ Calculation of Correction factor using Shear Condition



$\Sigma H = 0.$

$H_A + H_D = 100$ —— (1)

$\Sigma M_B = 0.$

$-H_A \times 6 + M_{AB} + M_{BA} = 0$

$6H_A = 8.6 + (3.59)K + 17.27 + (3.24)K.$

$6H_A = 25.87 + (6.83)K.$

$H_A = 4.31 + (1.13)k$

$\Sigma M_C = 0.$

$-H_D \times 4 + M_{CD} + M_{DC} = 0$

$H_D \times 4 = -22.45 + (4.81)k - 11.18 + (6.88)k$

$4H_D = -33.63 + (11.69)k.$

$H_D = -8.4 + (2.92)k$

① ⇒

$4.31 + (1.13)k - 8.4 + (2.92)k = 100$

$-4.09 + (4.05)k = 100$

$(4.05)k = 104.09$

$\boxed{k = 25.70}$

⑦ Final Moments.

$M_{AB} = 8.6 + 25.70(3.59) = 100.863$ kN-m

$M_{BA} = 17.27 + (3.24)25.7 = 100.538$ kN-m

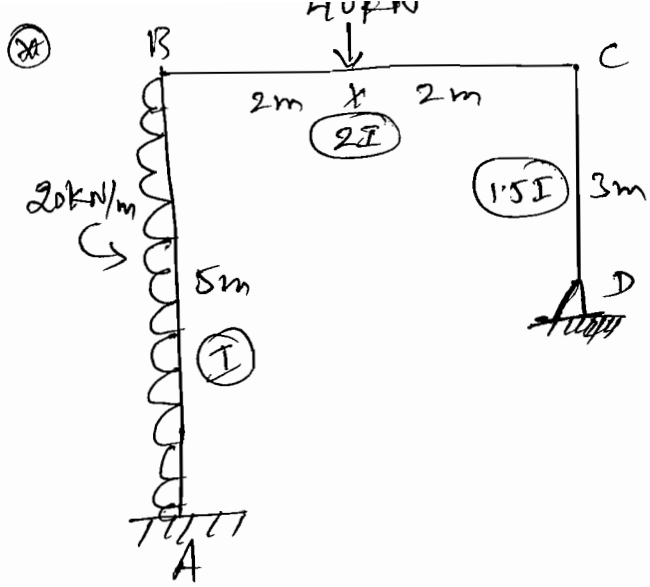
$M_{BC} = -17.27 - 3.24(25.7) = -100.538$ kN-m

$M_{CB} = 22.45 - 4.81(25.7) = -101.167$ kN-m

$M_{CD} = -22.45 + 4.81(25.7) = 101.167$ kN-m

$M_{DC} = -11.18 + 6.88(25.7) = 165.636$ kN-m

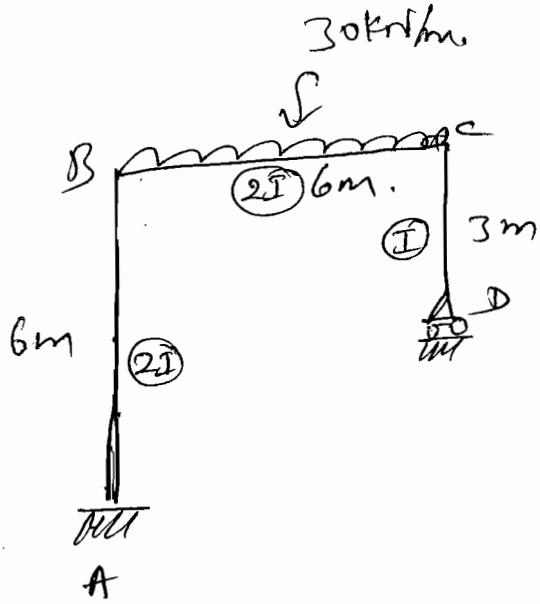
(*)



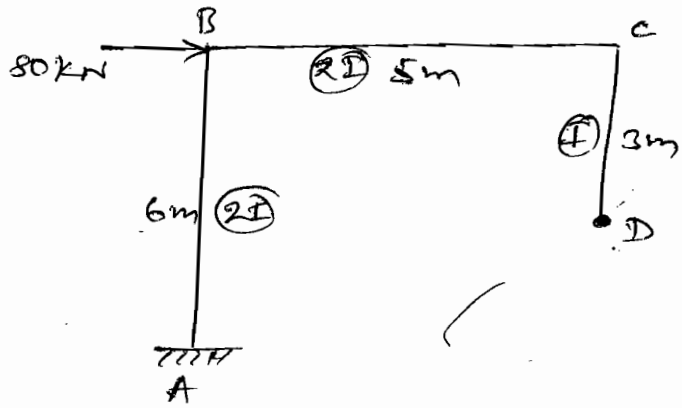
Ans:-

$\boxed{K = 1.022}$

(*)



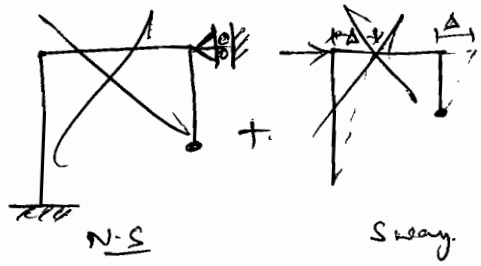
draw BMD & EC.



Sol

① FEM

All the FEM are zero.



② DF.

Joints	Members.	K	ΣK	DF.
B	BA	$2I/6 = 0.33I$.	$0.73I$	0.45
	BC	$2I/5 = 0.4I$.		0.55
C	CB	$2I/5 = 0.4I$.	$0.65I$	0.62
	CD	$3/4(I/3) = 0.25I$.		0.38

③ Non-sway Analysis

Since all the FEM are zero.

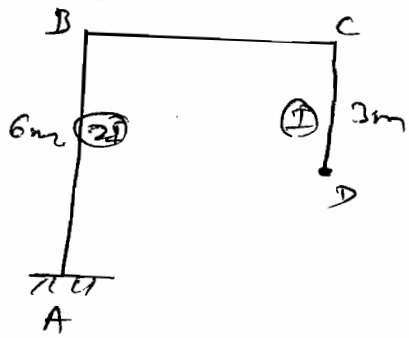
Non-sway analysis is not required. $(\bar{M}=0)$.

④ Sway Analysis.

$$M_{AB} = M_{BA} = \frac{-6EI\delta}{l^2}$$

$$= -\frac{6E(2I)\delta}{6^2}$$

$$= -\frac{EI\delta}{3}$$



$$M_{CD} = \frac{-3EI\delta}{l^2} = \frac{-3EI\delta}{3^2} = -\frac{EI\delta}{3}$$

$M_{DC} = 0.$ (Hinged).

$$\frac{M_{AB}}{M_{CD}} = \frac{M_{BA}}{M_{CD}} = \left(\frac{1}{1}\right)$$

∴ Assume suitable values according to above ratio.

$M_{AB} = M_{BA} = 5$ kN-m

$M_{CD} = 5$ kN-m.

$M_{DC} = 0$

A	B		C		D
	0.45	0.55	0.62	0.38	
5	5	0	0	5	0.
	-2.25	-2.75	-3.1	-1.9	
-1.125		-1.505	-1.375		
	0.67	0.827	0.85	0.52	
0.33		0.42	0.41		
	-0.19	-0.23	-0.25	-0.15	
-0.1		-0.12	-0.11		
	0.05	0.07	0.06	0.05	
4.105	3.28	-3.28	-3.52	+3.52	

⑤ Final Moments.

$M = \bar{M} + KM'$

$M = (\text{Nonsway}) + K(\text{Sway})$

$\bar{M} = 0.$

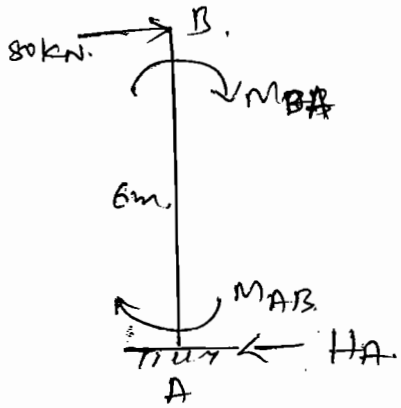
$M_{AB} = 4.105\,(k)$

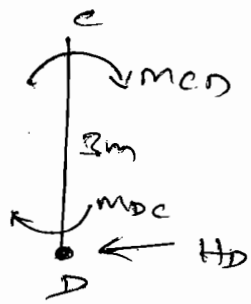
$$\left.\begin{aligned} M_{BA} &= 3.28\,(k) \\ M_{BC} &= -3.28\,(k) \\ M_{CB} &= -3.52\,(k) \\ M_{CD} &= 3.52\,(k) \end{aligned}\right\} \rightarrow \text{I}.$$

$M_{DC} = \underline{\underline{0.}}$

$$\frac{M_{BA}}{M_{CD}} = \frac{\frac{1}{2}EI\theta}{\frac{3}{8}EI\theta} = \frac{0.5EI\theta}{0.375EI\theta}$$

⑥ Shear Condition





$\Sigma H = 0.$ $\quad -H_A - H_D + 80 = 0.$

$H_A + H_D = 80. \longrightarrow$ ①

$\Sigma M_B = 0.$

$H_A \times 6 + M_{BA} + M_{AB} = 0$

$H_A = -\frac{1}{6}\left[(4.105)k + (3.28)k\right] = \underline{\underline{-1.23k.}}$

$\Sigma M_C = 0.$

$H_D \times 3 + M_{CD} = 0.$

$H_D = -\frac{1}{3}\left[3.52k\right] = \underline{\underline{-1.17k.}}$

① ⇒ $H_A + H_D = 80$

$-1.23k - 1.17k = 80$

$$\boxed{k = -33.33}$$

(7) Final Moments.

$M_{AB} = 4.105(-33.33) = -136.8$ kN-m

$M_{BA} = 3.28(-33.33) = -109.32$ kN-m

$M_{BC} = -3.28(-33.33) = 109.32$ kN-m

$M_{CB} = -3.52(-33.33) = 117.32$ kN-m

$M_{CD} = (3.52) - 33.33 = -117.32$ kN-m

$M_{DC} = 0.$

Brindavan College of Engineering, Bengaluru - 560 063

DEPARTMENT OF CIVIL ENGINEERING

INTERNAL ASSESSMENT TEST - I

ACADEMIC YEAR 2019 - 2020

USN										

Sem/Sec – 5th	**Subject code - 17CV52/15CV52**	**Subject name - ANALYSIS OF INDETERMINATE STRUCTURES**	**Duration -** *$1^1/_2$ hours*
Date - 17/10/2019			
Time – 2.00-3.30pm			**Max. Marks -** *50*

Course outcomes

CO2 - Determine the moment in indeterminate beams and frames of no sway and sway using moment distribution method.

CO3 - Construct the bending moment diagram for beams and frames by Kani's method.

Note: **i) *Answer any one full question from Q. No 1 & 2 and any one full question from Q. No 3 & 4.***

ii) All questions carry equal marks

Bloom's Knowledge level

L1 - *Remember*, L2 - *Understand*, L3 - *Apply*, L4 - *Analyze*, L5 - *Evaluate* & L6 - *Create*

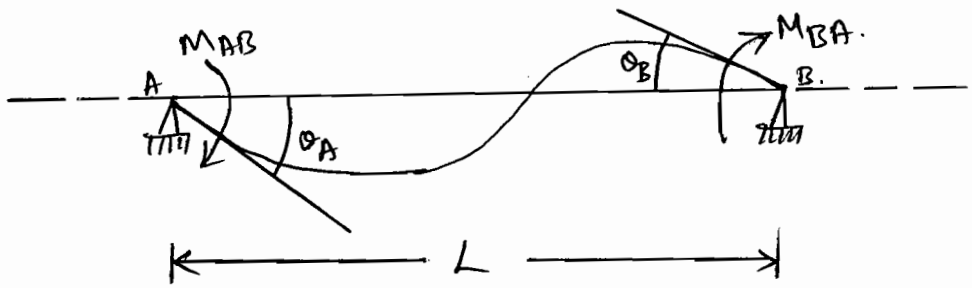
Syllabus - *Module 2 & 3*				
Q. No.	**Questions**	**Marks**	**Knowledge Level**	**COs**
1	Analyze the continuous beam shown in figure by Moment Distribution method. Draw BMD, SFD and Elastic curve.	25	L2,L4,L5	1
	OR			
2	Analyze the continuous beam shown in figure by Moment Distribution method, if support B sinks by 12mm. Given E = 200kN/mm^2 and I = 20X10^{6}mm^4 by. Draw BMD, SFD and Elastic curve.	25	L2,L4,L5	1
3	Analyze the frame shown in figure by Moment Distribution method. Draw BMD, SFD and Elastic curve.	25	L2,L4,L5	1
	OR			
4	Analyze the continuous beam shown in figure by Kani's method. Draw BMD, SFD and Elastic curve.	25	L2,L4,L5	1

Faculty in charge

HOD

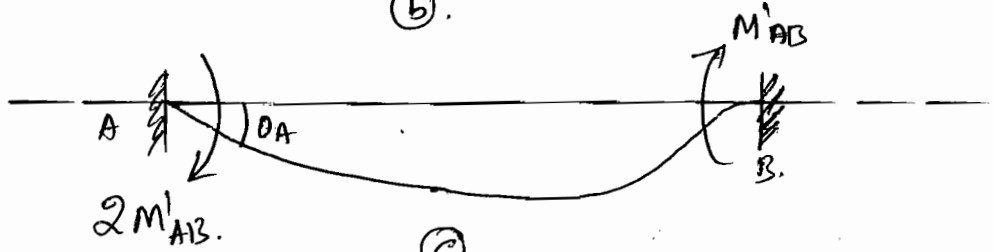
KANI'S METHOD.

Analysis of Structures without Relative Displacement at ends.

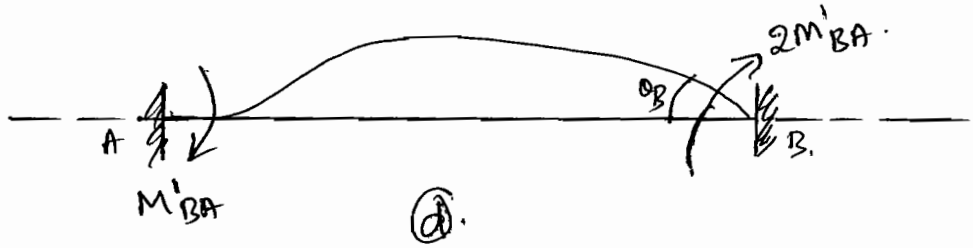


ⓐ Typical Member.

ⓑ.



ⓒ.



ⓓ.

Members AB shown in fig ⓐ is an intermediate member of a beam/frame, which has no relative displacements at the ends (ie ends A & B are at the same level).

Let M_{AB} & M_{BA} be the final end moments.

M_{AB} may Consists of.

(i) Fixed End Moments (θ_A & $\theta_B = 0$), fig (b)

(ii) Moment due to rotation of end 'A' only (fig (c))

(iii) Moment due to rotation of end 'B' only (fig (d)).

(1) Rotation Factor.

$$\text{Rotation Factor (R.F)} \; (U) = \frac{-1}{2}\left(\frac{K}{\Sigma K}\right).$$

(2) Rotation Moment.

$$M'_{ab} = U\left[\Sigma M_F + M'_{ba}\right]$$

$$M'_{ba} = U\left[\Sigma M_F + M'_{ab}\right]$$

(3) Final Moment.

$$M_{AB} = M_{FAB} + 2M'_{ab} + M'_{ba}.$$

$$M_{BA} = M_{FBA} + 2M'_{ba} + M'_{ab}.$$

(4) Additional Moment.

(a) Due to Sinking, $m = \dfrac{-6EI\Delta}{l^2}.$

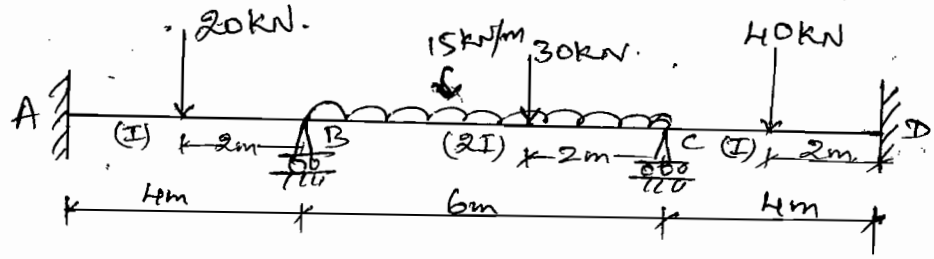
(b) Due to Rotation,

(i) Near End, $m = \dfrac{4EI\theta}{l}.$

(ii) Far End, $m = \dfrac{2EI\theta}{l}.$

Analysis of Continuous Beams.

1) Analyse the Continuous Beam shown in figure by Kani's Method.



Sol

(i) FEM

$$M_{FAB} = -\frac{Wl}{8} = -\frac{20\times4}{8} = -10\,kN\text{-}m$$

$$M_{FBA} = +\frac{Wl}{8} = 10\,kN\text{-}m$$

$$M_{FBC} = -\frac{Wl^2}{12} - \frac{Wab^2}{l^2} = -\frac{15\times6^2}{12} - \frac{30\times4\times2^2}{6^2} = -58.33\,kN\text{-}m$$

$$M_{FCB} = +\frac{Wl^2}{12} + \frac{Wa^2b}{l^2} = 71.67\,kN\text{-}m = \frac{15\times6^2}{12} + \frac{30\times4^2\times2}{6^2}$$

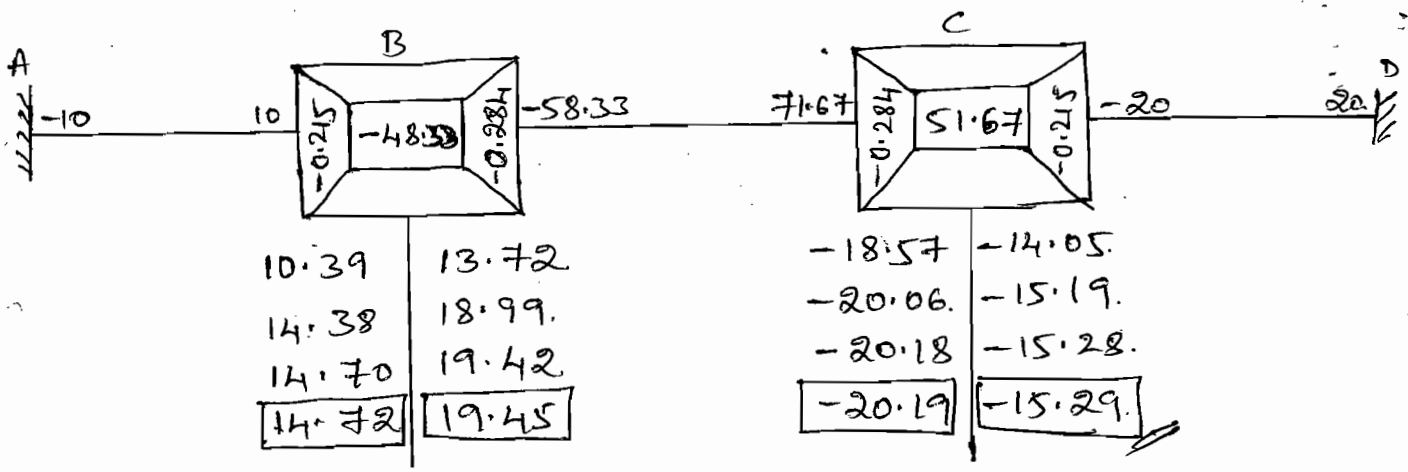
$$M_{FCD} = -\frac{Wl}{8} = -\frac{40\times4}{8} = -20\,kN\text{-}m$$

$$M_{FDC} = +\frac{Wl}{8} = 20\,kN\text{-}m$$

(ii) Rotation factors. $RF = -\frac{1}{2}\left(K/\Sigma K\right)$

Joint	Members.	K	Σk	RF.
B	BA	$\frac{I}{L} = \frac{I}{4} = 0.25I.$	0.58I	−0.215
	BC	$\frac{I}{L} = \frac{2I}{6} = 0.33I.$		−0.284
C	CB	$\frac{I}{L} = \frac{2I}{6} = 0.33I.$	0.58I	−0.284
	CD	$\frac{I}{L} = \frac{I}{4} = 0.25I$		−0.215.

(iii) Rotation Moment. $M'_{ba} = U[M_F + M'_{ba}]$



Cycle - I.

$M'_{ba} = -0.215[-48.33 + 0] = 10.39$

$M'_{bc} = -0.284[-48.33 + 0] = 13.72$

$M'_{cb} = -0.284[51.67 + 13.72] = -18.57$

$M'_{cd} = -0.215[51.67 + 13.72] = -14.05$

Cycle - II.

$M'_{ba} = -0.215[-48.33 - 18.57] = 14.38$

$M'_{bc} = -0.284[-48.33 - 18.57] = 18.99$

$M'_{cb} = -0.284[51.67 + 18.99] = -20.06$

$M'_{cd} = -0.215[51.67 + 18.99] = -15.19$

Cycle III

$M'_{ba} = -0.215[-48.33 - 20.06] = \cancel{19.42}\ 14.70$

$M'_{bc} = -0.284[-48.33 - 20.06] = 19.42$

$M'_{cb} = -0.284[+51.67 + 19.42] = -20.18$

$M'_{cd} = -0.215[51.67 + 19.42] = -15.28$

$M'_{ba} = -0.215[-48.33 - 20.18] = 14.72$

$M'_{bc} = -0.284[\cancel{51.67}\ -48.33 - 20.18] = 19.45$

$M'_{cb} = -0.284[51.67 + 19.45] = -20.19$

$M'_{cd} = -0.215[51.67 + 19.45] = -15.29$

(iv) Final Moment.

$$\boxed{M_{AB} = M_F + 2M'_{ab} + M'_{ba}}$$

$M_{AB} = -10 + (2\times 0) + 14.72 = 4.72$ kN-m

$M_{BA} = +10 + (2\times 14.72) + 0 = 39.44$ kN-m

$M_{BC} = -58.33 + 2(19.45) - 20.19 = -39.62$ kN-m

$M_{CB} = 71.67 + 2(-20.19) + 19.45 = 50.74$ kN-m

$M_{CD} = -20 + 2(-15.29) + 0 = -50.58$ kN-m

$M_{DC} = 20 + 2(0) - 15.29 = 4.71$ kN-m

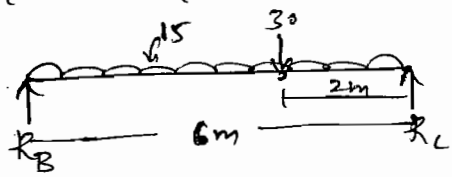
(65×2)

$-(15\times 2\times \frac{2}{2})$

$BM)_{AB} = \frac{Wl}{4} = \frac{20\times 4}{4} = 20$ kN-m

$BM)_{CD} = \frac{Wl}{4} = \frac{40\times 4}{4} = 40$ kN-m

$BM)_{BC}$



$\Sigma V = 0$

$R_B + R_C = (15\times 6) + 30 = 120.$

$\Sigma M_B = 0.$

$-R_C\times 6 + 30\times 4 + 15\times 6\times \frac{6}{2} = 0.$

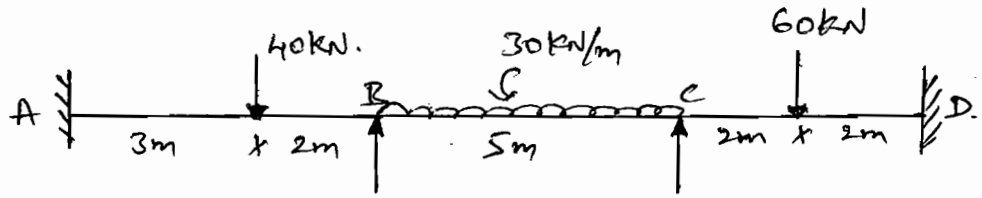
$R_C = 65$ kN.

$R_B = 120 - 65 = 55$ kN.

$M_E = (65\times 2) - (15\times 2\times \frac{2}{2})$

$= 100$ kN-m.

2) Analyse the C.B shown in figure by Kani's method.



Sol:-

(i) FEM.

$$M_{FAB} = \frac{-Wab^2}{l^2} = \frac{-40 \times 3 \times 2^2}{5^2} = -19.2 \text{ kN-m}$$

$$M_{FBA} = \frac{Wa^2b}{l^2} = \frac{40 \times 3^2 \times 2}{5^2} = 28.8 \text{ kN-m}$$

$$M_{FBC} = \frac{-Wl^2}{12} = -62.5 \text{ kN-m}$$

$$M_{FCB} = \frac{+Wl^2}{12} = 62.5 \text{ kN-m}$$

$$M_{FCD} = \frac{-Wl}{8} = \frac{-60 \times 4}{8} = -30 \text{ kN-m}$$

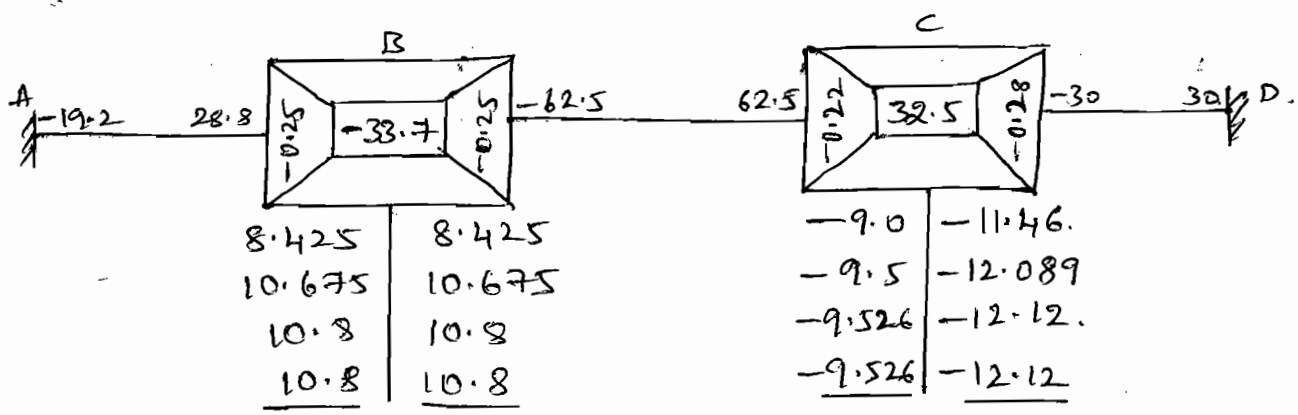
$$M_{FDC} = \frac{Wl}{8} = 30 \text{ kN-m}$$

(ii) Rotation factor.

Joint	Members	k	Σk	$RF = \frac{-1}{2}\left(\frac{k}{\Sigma k}\right)$
B	BA	$I/L = I/5 = 0.2I$.	$0.4I$.	-0.25
	BC	$I/L = I/5 = 0.2I$.		-0.25
C	CB	$I/L = I/5 = 0.2I$.	$0.45I$	-0.22
	CD	$I/L = I/4 = 0.25I$		-0.28.

(iii) Rotation Moment.

$$M'_{bc} = U\left[M_F + M'_{cb}\right].$$



(iv) Final Moment.

$$M_{AB} = M_F + 2M'_{ab} + M'_{ba}$$

$$= -19.2 + (2\times 0) + 10.8 = -8.4 \text{ kN-m}$$

$$M_{BA} = 28.8 + (2\times 10.8) + 0 = 50.4 \text{ kN-m}$$

$$M_{BC} = -62.5 + 2(10.8) - 9.526 = -50.42 \text{ kN-m}$$

$$M_{CB} = +62.5 + (2\times -9.526) + 10.8 = 54.24 \text{ kN-m}$$

$$M_{CD} = -30 + 2(-12.12) + 0 = -54.24 \text{ kN-m}$$

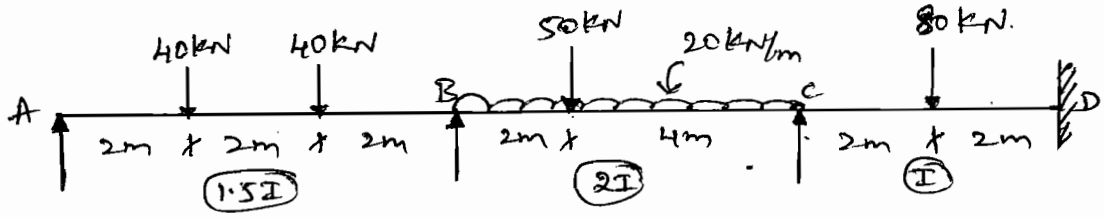
$$M_{DC} = 30 + (2\times 0) - 12.12 = 17.88 \text{ kN-m}$$

$$BM)_{AB} = \frac{Wab}{l} = \frac{40\times 3\times 2}{5} = 48 \text{ kN-m}$$

$$BM)_{BC} = \frac{Wl^2}{8} = \frac{30\times 5^2}{8} = 93.75 \text{ kN-m}$$

$$BM)_{CD} = \frac{Wl}{4} = \frac{60\times 4}{4} = 60 \text{ kN-m}$$

3) Analyse the CB shown in figure by Kani's method.



Sol:-

(i) FEM.

$$M_{FAB} = -\frac{Wab^2}{l^2} = -\frac{40\times2\times4^2}{6^2} - \frac{40\times4\times2^2}{6^2} = -53.33\ kN\text{-}m$$

$$M_{FBA} = \frac{Wa^2b}{l^2} = \frac{40\times2\times4^2}{6^2} + \frac{40\times4\times2^2}{6^2} = 53.33\ kN\text{-}m$$

$$M_{FBC} = \frac{-Wl^2}{12} - \frac{Wab^2}{l^2} = -\frac{20\times6^2}{12} - \frac{50\times2\times4^2}{6^2} = -104.44\ kN\text{-}m$$

$$M_{FCB} = +\frac{Wl^2}{12} + \frac{Wa^2b}{l^2} = 82.22\ kN\text{-}m$$

$$M_{FCD} = \frac{-Wl}{8} = -\frac{80\times4}{8} = -40\ kN\text{-}m$$

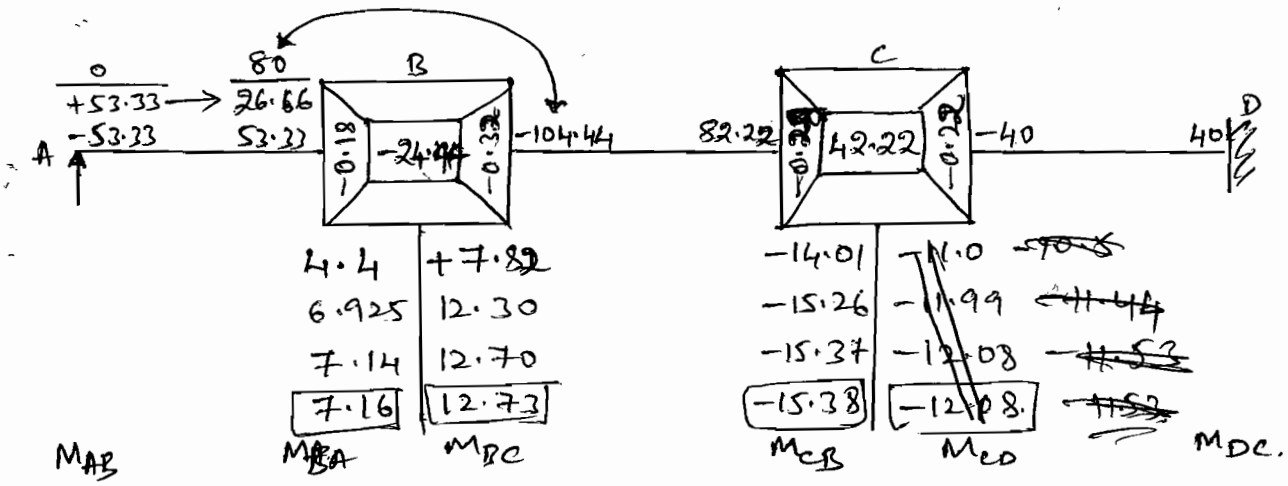
$$M_{FDC} = \frac{Wl}{8} = 40\ kN\text{-}m$$

(ii) Rotation factor.

Joint	Members	K	ΣK	$RF = \frac{-1}{2}\left(\frac{K}{\Sigma K}\right)$
B	BA	$\frac{3}{4}(I/L) = \frac{3}{4}\left(\frac{1.5I}{6}\right) = 0.1875I$	0.5205I	−0.18
	BC	$I/L = \frac{2I}{6} = 0.333I$		−0.32
C	CB	$I/L = \frac{2I}{6} = 0.333I$	0.583I	−0.28
	CD	$I/L = I/4 = 0.25I$		−0.22

(iii) Rotation Moment

$$M'_{bc} = U\left[M_F + M'_{cb}\right]$$



(iv) Final Moments.

$$M_{AB} = M_F + 2M'_{ab} + M'_{ba}$$

$M_{AB} = 0.$

$M_{BA} = 80 + 2(7.16) = 94.32$ kN-m ↻

$M_{BC} = -104.4 + 2(12.73) - 15.38 = -94.32$ kN-m ↺

$M_{CB} = 82.22 + 2(-15.38) + 12.73 = 64.20$ kN-m ↻

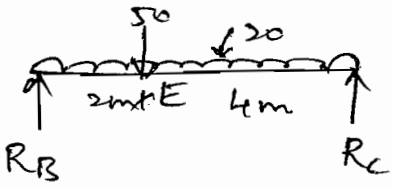
$M_{CD} = -40 + 2(-12.08) + 0 = -64.16$ kN-m ↺

$M_{DC} = 40 + 2(0) + (-12.08) = 27.92$ kN-m ↻

$BM)_{AB} = W.a = 40 \times 2 = 80$ kN-m

$BM)_{CD} = \frac{Wl}{4} = \frac{80 \times 4}{4} = 80$ kN-m

$BM)_{BC} \Rightarrow$



$\Sigma V = 0$

$R_B + R_c = 50 + (20 \times 6) = 170$

$\Sigma M_B = 0$

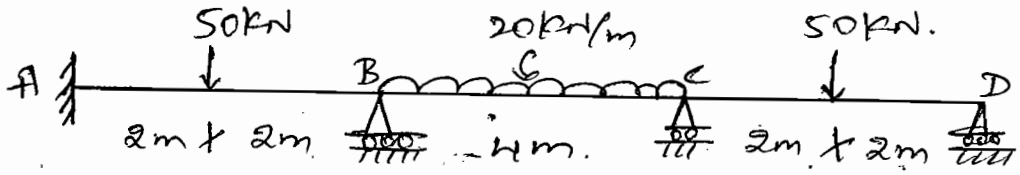
$50 \times 2 + 20 \times 6 \times \frac{6}{2} - R_c \times 6 = 0$

$R_c = 76.67$ kN.

$M)_E = (93.33 \times 2) - \left(20 \times 2 \times \frac{2}{2}\right)$

$= 146.66$ kN-m

(4) Analyse the CB shown in figure by Kani's Method.



Sol:-

(i) FEM.

$$M_{FAB} = \frac{-Wl}{8} = \frac{-50\times4}{8} = -25 \text{ kN-m}$$

$$M_{FBA} = +\frac{Wl}{8} = 25 \text{ kN-m}$$

$$M_{FBC} = \frac{-Wl^2}{12} = \frac{-20\times4^2}{12} = -26.67 \text{ kN-m}$$

$$M_{FCB} = +\frac{Wl^2}{12} = 26.67 \text{ kN-m}$$

$$M_{FCD} = -\frac{Wl}{8} = -25 \text{ kN-m}$$

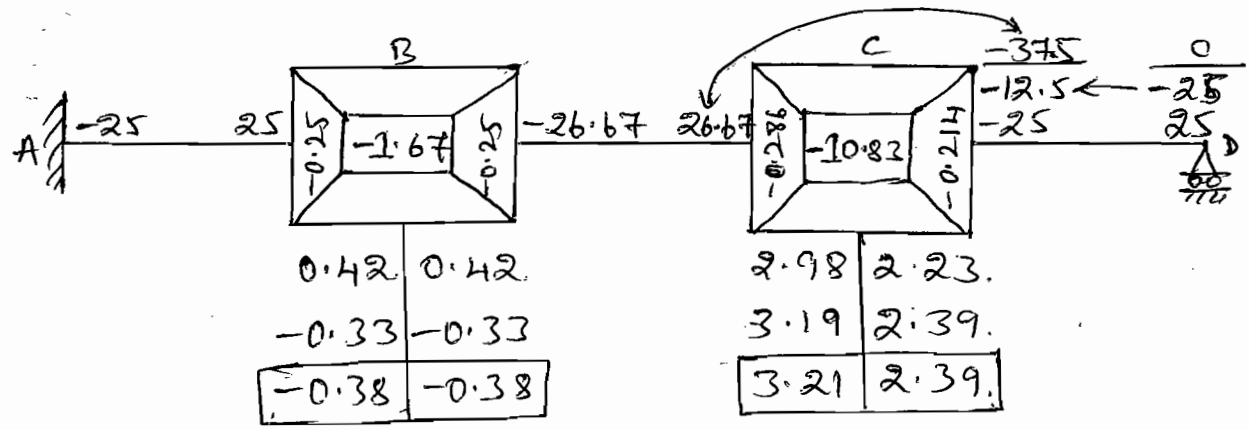
$$M_{FDC} = \frac{Wl}{8} = 25 \text{ kN-m}$$

(ii) Rotation factors.

Joints	Members	K	ΣK	RF(U) $= -\frac{1}{2}\left(K/\Sigma K\right)$
B	BA	$I/L = I/4 = 0.25I$	0.5I	−0.25
	BC	$I/L = I/4 = 0.25I$		−0.25
C	CB	$I/L = I/4 = 0.25I$	0.4375I	−0.286
	CD	$\frac{3}{4}\left(\frac{I}{L}\right) = \frac{3}{4}\left(\frac{I}{4}\right) = 0.1875I$		−0.214

(iii) Rotation Moment.

$$M'_{bc} = U\left[\Sigma M_F + \Sigma M'_{cb}\right]$$



(iv) Final Moments.

$$M_{AB} = M_F + 2M'_{ab} + M'_{ba}$$

$$M_{AB} = -25 + 2(0) - 0.38 = -25.38 \text{ kN-m}$$

$$M_{BA} = 25 + 2(-0.38) + 0 = 24.24 \text{ kN-m}$$

$$M_{BC} = -26.67 + 2(-0.38) + 3.21 = -24.22 \text{ kN-m}$$

$$M_{CB} = 26.67 + 2(3.21) - 0.38 = 32.71 \text{ kN-m}$$

$$M_{CD} = -37.5 + 2(2.39) + 0 = -32.72 \text{ kN-m}$$

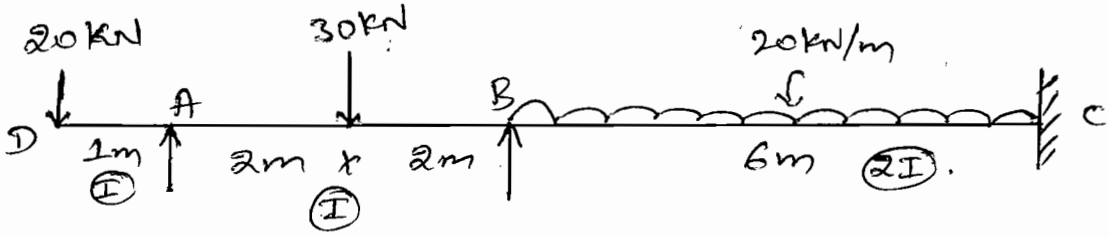
$$M_{DC} = 0.$$

$$BM)_{AB} = \frac{Wl}{4} = \frac{50 \times 4}{4} = 50 \text{ kN-m}$$

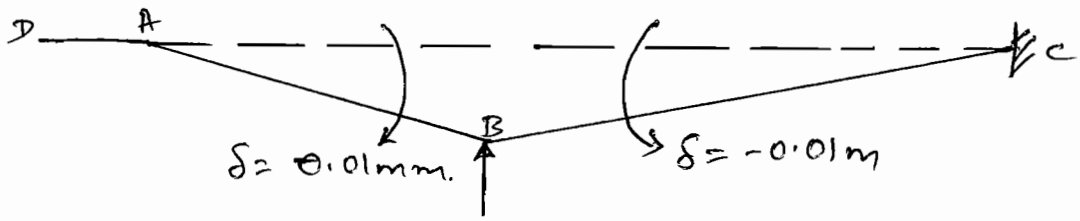
$$BM)_{BC} = \frac{Wl^2}{8} = \frac{20 \times 4^2}{8} = 40 \text{ kN-m}$$

$$BM)_{CD} = \frac{Wl}{4} = \frac{50 \times 4}{4} = 50 \text{ kN-m}$$

5) Analyse the CB shown in figure by Kani's Method. The support B sinks by 10mm. Take EI = 4000 kN-m². Draw BMD & elastic curve



Sol:



Additional moment due to sinking $= -\frac{6EI\delta}{l^2}$

(i) FEM.

$M_{AD} = +20\times1 = 20$ kN-m (Final overhanging moment)

$$M_{FAB} = -\frac{Wl}{8} - \frac{6EI\delta}{l^2} = -\frac{30\times4}{8} - \frac{6\times4000\times(+0.01)}{4^2}$$

$$= -30 \text{ kN-m}$$

$$M_{FBA} = \frac{Wl}{8} - \frac{6EI\delta}{l^2} = 0.$$

$$M_{FBC} = -\frac{Wl^2}{12} - \frac{6EI\delta}{l^2} = \frac{-20\times6^2}{12} - \frac{6\times4000\times(-0.01)\times2}{6^2}$$

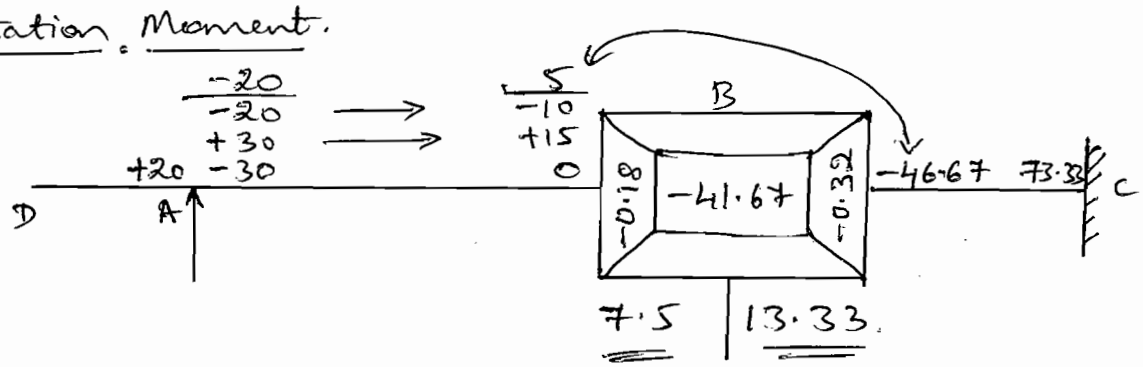
$$= -46.67 \text{ kN-m}$$

$$M_{FCB} = \frac{Wl^2}{12} - \frac{6EI\delta}{l^2} = 73.33 \text{ kN-m}$$

(ii) Rotation Factor:

Joint	Members	K	ΣK	$RF = -\frac{1}{2}(K/\Sigma K)$
B	BA	$\frac{3}{4}(I/L) = 0.1875I$	$0.5205I$	-0.18
	BC	$I/L = \frac{2I}{6} = 0.333I$		-0.32

(iii) Rotation Moment.



(iv) Final Moments.

$$M_{AB} = M_F + 2(M'_{ab}) + M'_{ba}$$

$M_{AD} = 20$ kN-m

$M_{AB} = -20$ kN-m

$M_{BA} = 5 + 2(7.5) + 0 = 20$ kN-m

$M_{BC} = -46.67 + 2(13.33) + 0 = -20$ kN-m

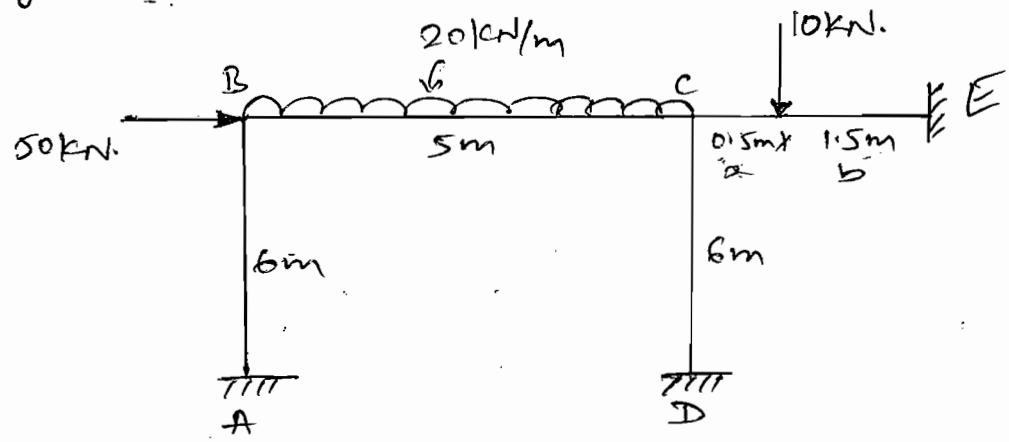
$M_{CB} = 73.33 + (2 \times 0) + 13.33 = 86.66$ kN-m

$BM)_{AB} = \frac{Wl}{4} = \frac{30 \times 4}{4} = 30$ kN-m

$BM)_{BC} = \frac{Wl^2}{8} = \frac{20 \times 6^2}{8} = 90$ kN-m

Analysis of Non-Sway Frames.

1) Analyse the frame shown in figure by Kani's Method.



Sol:- (i) FEM.

$$M_{FAB} = M_{FBA} = M_{FCD} = M_{FDC} = 0.$$

$$M_{FBC} = -\frac{wl^2}{12} = -\frac{20\times5^2}{12} = -41.67 \text{ kN-m}$$

$$M_{FCB} = +\frac{wl^2}{12} = 41.67 \text{ kN-m}$$

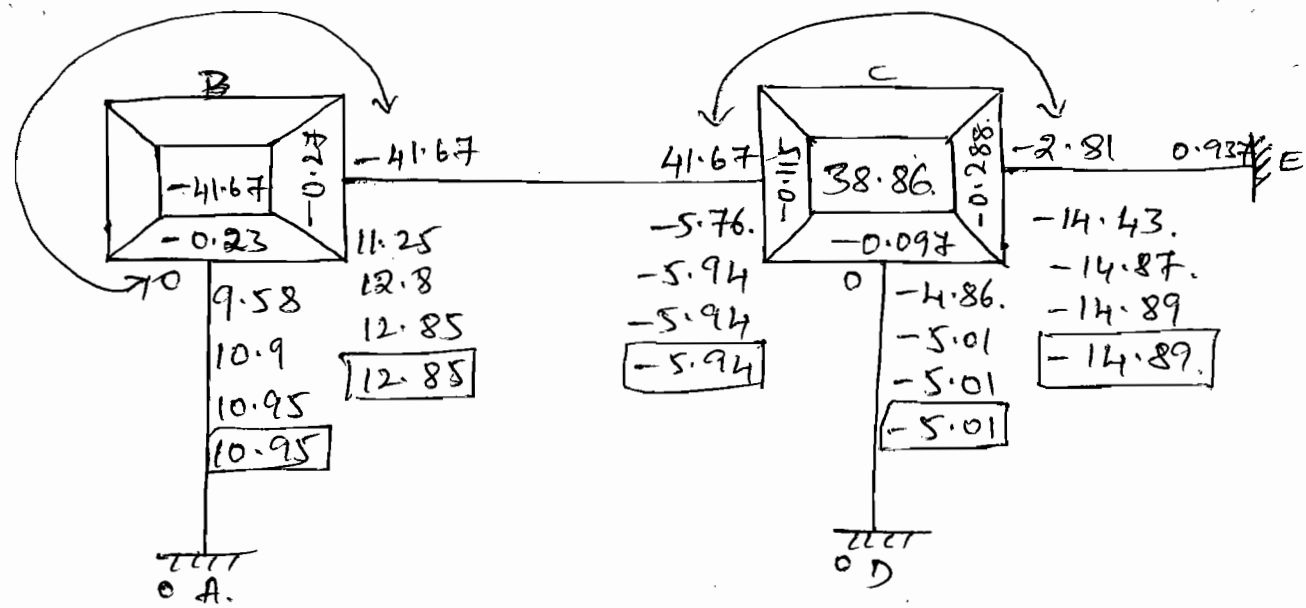
$$M_{FCE} = -\frac{wab^2}{l^2} = -2.81 \text{ kN-m}$$

$$M_{FEC} = +\frac{wa^2b}{l^2} = 0.937 \text{ kN-m}$$

(ii) Rotation factors.

Joints	Members	k	Σk	$RF = -\frac{1}{2}(k/\Sigma k)$
B	BA	$I/L = I/6 = 0.167I$	$0.367I$	-0.23
	BC	$I/L = I/5 = 0.2I$		-0.27
C	CB	$I/L = I/5 = 0.2I$	$0.867I$	-0.115
	CE	$I/L = I/2 = 0.5I$		-0.288
	CD	$I/L = I/6 = 0.167I$		-0.097

(ii) Iteration Moments.



(iv) Final Moments.

$M^*_{AB} = 0 + 2(0) + 10.95 = 10.95$ kN-m

$M_{BA} = 0 + 2(10.95) + 0 = 21.9$ kN-m

$M_{BC} = -41.67 + 2(12.85) + (-5.94) = -21.91$ kN-m

$M_{CB} = 41.67 + 2(-5.94) + 12.85 = 42.64$ kN-m

$M_{CE} = \cancel{-0.09} -2.81 + 2(-14.89) + 0 = -32.59$ kN-m

$M_{CD} = 0 + 2(-5.01) + 0 = -10.2$ kN-m

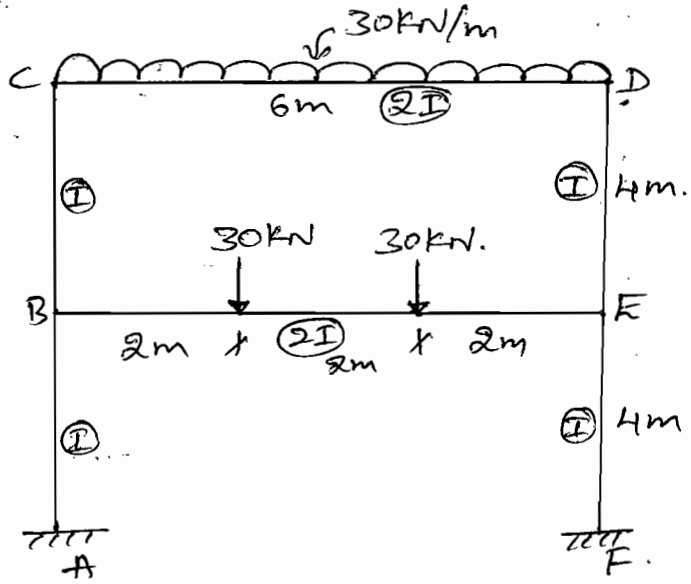
$M_{EC} = 0.937 + (2\times 0) - 14.89 = -13.95$ kN-m

$M_{DC} = 0 + 2(0) - 5.01 = -5.01$ kN-m

$BM)_{BC} = \frac{wl^2}{8} = \frac{20\times 5^2}{8} = 62.5$ kN-m

$BM)_{CE} = \frac{Wab}{l} = \frac{10\times 0.5\times 1.5}{2} = 3.75$ kN-m

(2) Analyse the frame shown in figure by Kani's method.



Sol:-

(i) FEM

$M_{FAB} = M_{FBA} = M_{FBC} = M_{FCB} = 0.$

$M_{FDE} = M_{FED} = M_{FEF} = M_{FFE} = 0.$

$$M_{FBE} = -\frac{Wab^2}{l^2} = -\left[\frac{30\times 2\times 4^2}{6^2} + \frac{30\times 4\times 2^2}{6^2}\right]$$

$$= -40\,kN\text{-}m$$

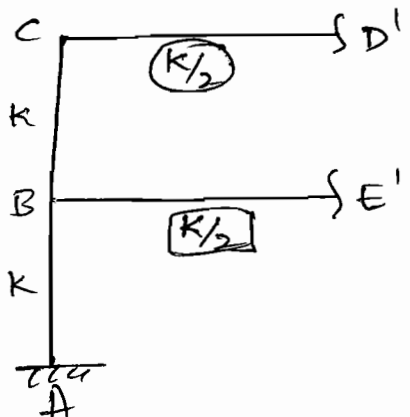
$$M_{FEB} = +\frac{Wa^2b}{l^2} = \left[\frac{30\times 2^2\times 4}{6^2} + \frac{30\times 4^2\times 2}{6^2}\right]$$

$$= +40\,kN\text{-}m$$

$$M_{FCD} = -\frac{Wl^2}{12} = -\frac{30\times 6^2}{12} = -90\,kN\text{-}m$$

$$M_{FDC} = +Wl^2/12 = 90\,kN\text{-}m$$

(ii) ~~Rotation moment.~~



(iv) Final Moments.

$M_{AB} = 0 + 2(0) + 2.57 = 2.57$ kN-m

$M_{BA} = 0 + (2 \times 2.57) + 0 = 5.14$ kN-m

$M_{BC} = 0 + 2(2.57) + 26.22 = 31.36$ kN-m

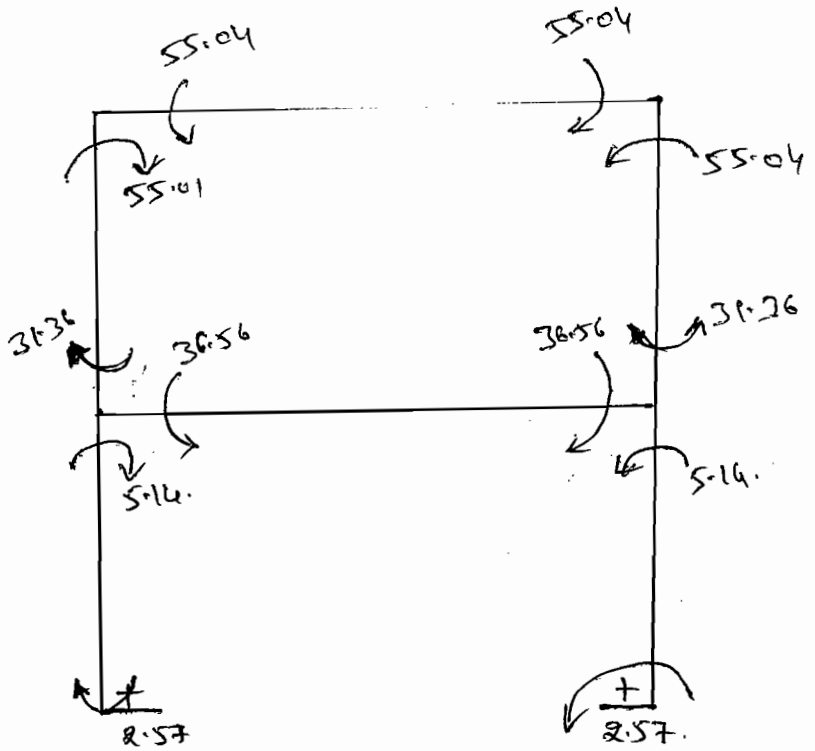
$M_{BE} = -40 + 2(1.72) + 0 = -36.56$ kN-m

$M_{CB} = 0 + 2(26.22) + 2.57 = 55.01$ kN-m

$M_{CD} = -90 + 2(17.48) + 0 = -55.04$ kN-m

$BM)_{BE} = W \times a = 30 \times 2 = 60$ kN-m

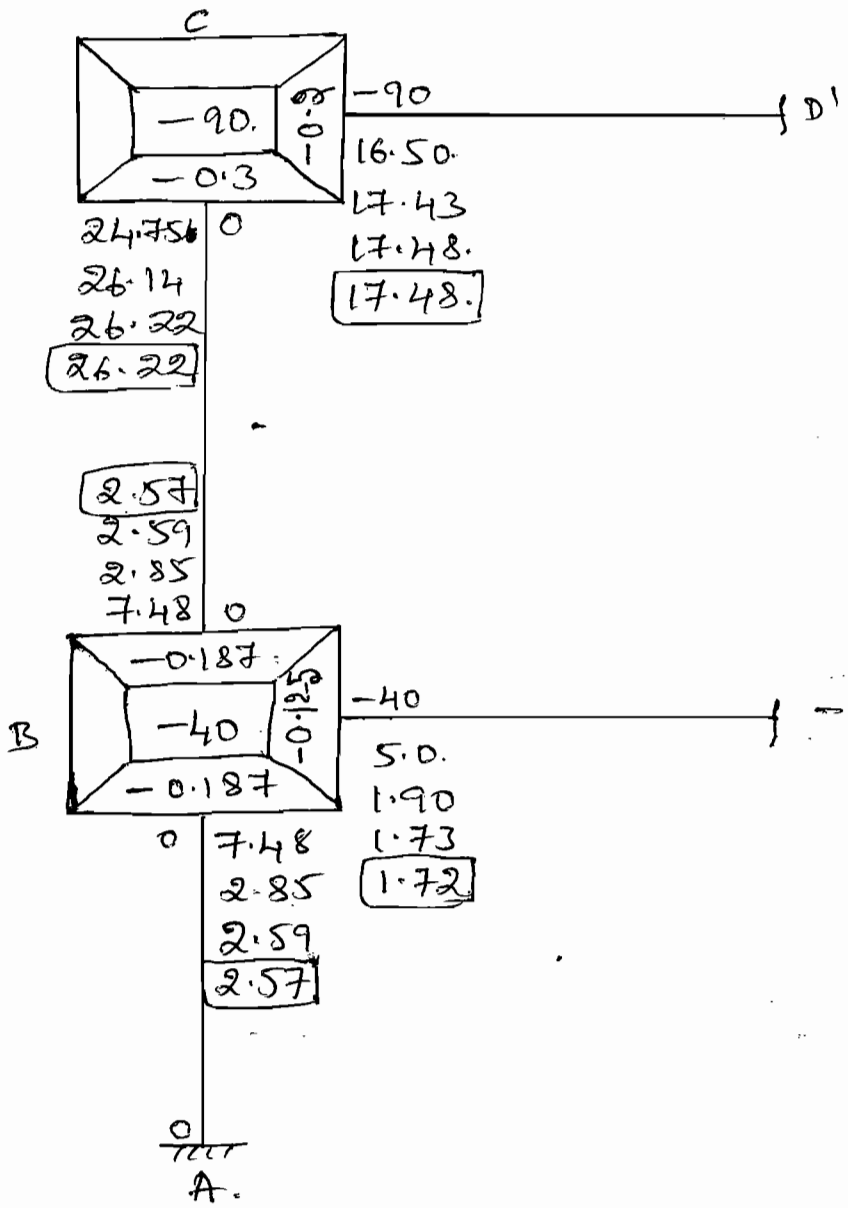
$BM)_{CD} = \frac{wl^2}{8} = 135$ kN-m



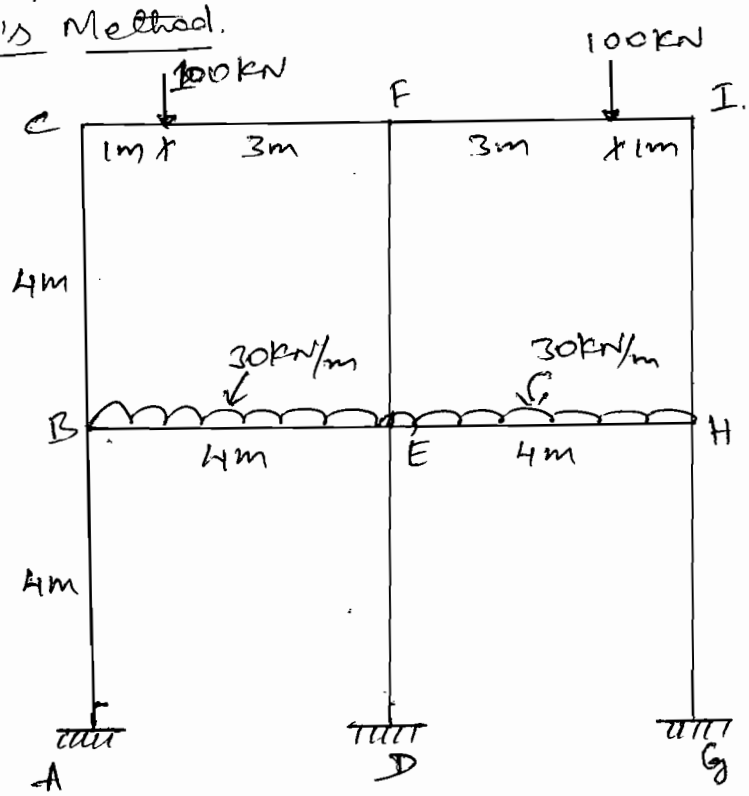
(ii) Rotation Factor

Joints	Members.	K	ΣK	$RF = -\frac{1}{2}(K/\Sigma K)$
B	BA	$I/L = I/4 = 0.25I$	$0.667I$	-0.187
	BC	$I/L = I/4 = 0.25I$		-0.187
	BE	$(K/2) = \frac{1}{2}\left(\frac{2I}{6}\right) = 0.167I$		-0.125
C	CB	$I/L = I/4 = 0.25I$	$0.417I$	-0.30
	CD	$\frac{1}{2}(K) = \frac{1}{2}\left(\frac{2I}{6}\right) = 0.167I$		-0.2

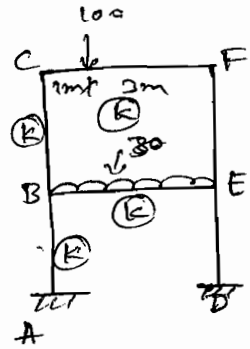
(iii) Rotation Moment.



③ Analyse the frame shown in figure by Kani's Method.



Sol:



(i) FEM

$$M_{FBE} = \frac{-wl^2}{12} = \frac{-30\times4^2}{12} = -40\,kN\text{-}m$$

$$M_{FEB} = \frac{+wl^2}{12} = 40\,kN\text{-}m$$

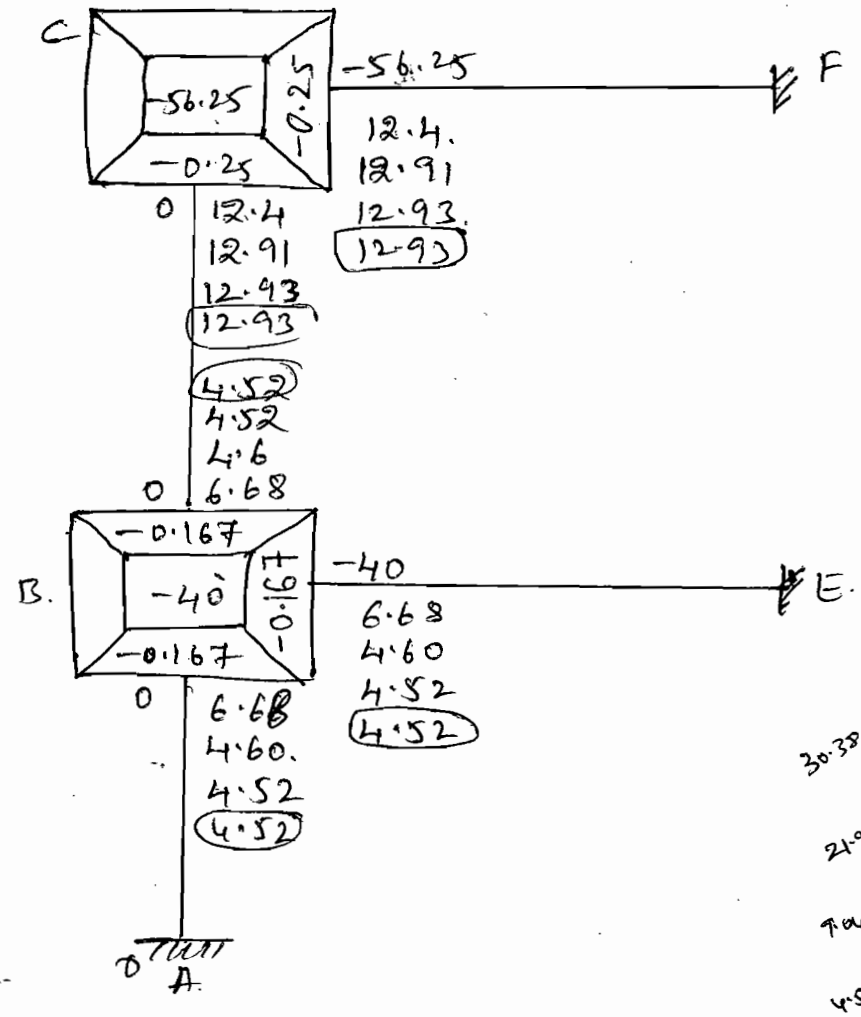
$$M_{FCF} = \frac{-wab^2}{l^2} = -56.25\,kN\text{-}m$$

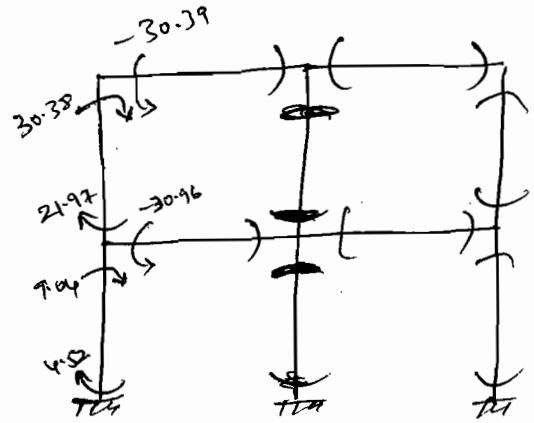
$$M_{FFC} = \frac{+wa^2b}{l^2} = 18.75\,kN\text{-}m$$

(ii) Rotation factors.

Joint	Member	k	Σk	RF = $-\frac{1}{2}(k/\Sigma k)$
B	BA	$I/4 = 0.25I$	$0.75I$	-0.167
	BC	$I/L = I/4 = 0.25I$		-0.167
	BE	$I/L = I/4 = 0.25I$		-0.167
C	CB	$I/4 = 0.25I$	$0.5I$	-0.25
	CD	$I/4 = 0.25I$		-0.25

(iii) Rotation Moment





(iv) Final Moments.

$M_{AB} = 0 + 2(0) + 4.52 = 4.52$ kN-m

$M_{BA} = 0 + 2(4.52) + 0 = 9.04$ —"—

$M_{BC} = 0 + 2(4.52) + 12.93 = 21.97$ kN-m

$M_{BE} = -40 + 2(4.52) + \cancel{4.52}\ 0 = -30.96$ kN-m

$M_{CB} = 0 + 2(12.93) + 4.52 = 30.38$ kN-m

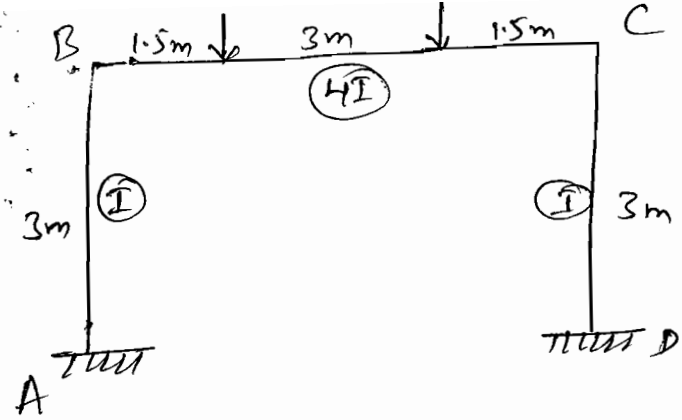
$M_{CF} = -56.25 + 2(12.93) + 0 = -30.39$ kN-m

$M_{FC} = 18.75 + 2(0)$
$= 31.68 + 12.93$ kN-m

$M_{EB} = 40 + 2(0) + 4.52$
$= 44.52$ kN-m

$BM)_{CF} = \frac{Wab}{l} = \frac{100 \times 1 \times 3}{4} = 75$ kN-m

$BM)_{BE} = \frac{Wl^2}{8} = \frac{30 \times 4^2}{8} = 60$ kN-m



Sol: ① FEM.

$$M_{FBC} = -\frac{Wab^2}{l^2} = -\left(\frac{40\times1.5\times4.5^2}{6^2} + \frac{40\times4.5\times1.5^2}{6^2}\right)$$

$$= -45\ \text{kN-m}$$

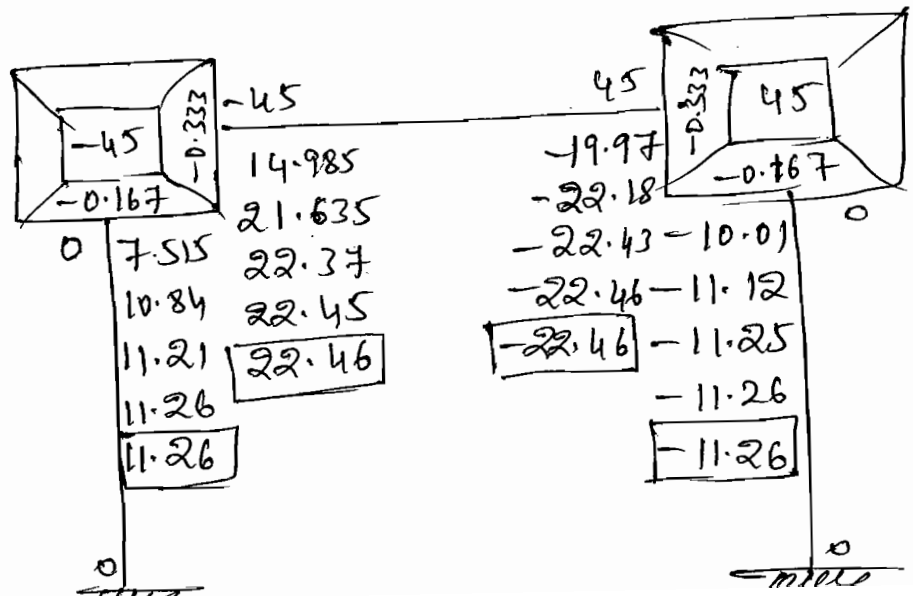
$$M_{FCB} = +\frac{Wa^2b}{l^2} = +\left(\frac{40\times1.5^2\times4.5}{6^2} + \frac{40\times4.5^2\times1.5}{6^2}\right)$$

$$= 45\ \text{kN-m}$$

② R.F.

Joints	Members	k	Σk	RF = $-\frac{1}{2}\left(\frac{k}{\Sigma k}\right)$
B	BA	$I/3 = 0.33I$	$0.99I$	-0.167
	BC	$4I/6 = 0.66I$		-0.333
C	CB	$4I/6 = 0.66I$	0.99	-0.333
	CD	$I/3 = 0.33I$		-0.167

③ R.M.



$M_{AB} = 0 + 2(0) + 11.26 = 11.26$ kN-m

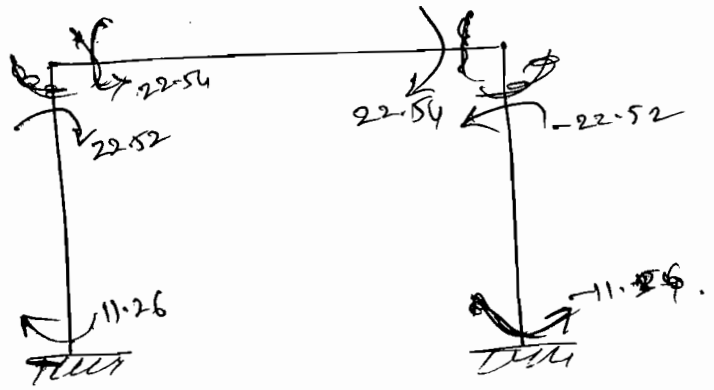
$M_{BA} = 0 + 2(11.26) + 0 = 22.52$ kN-m

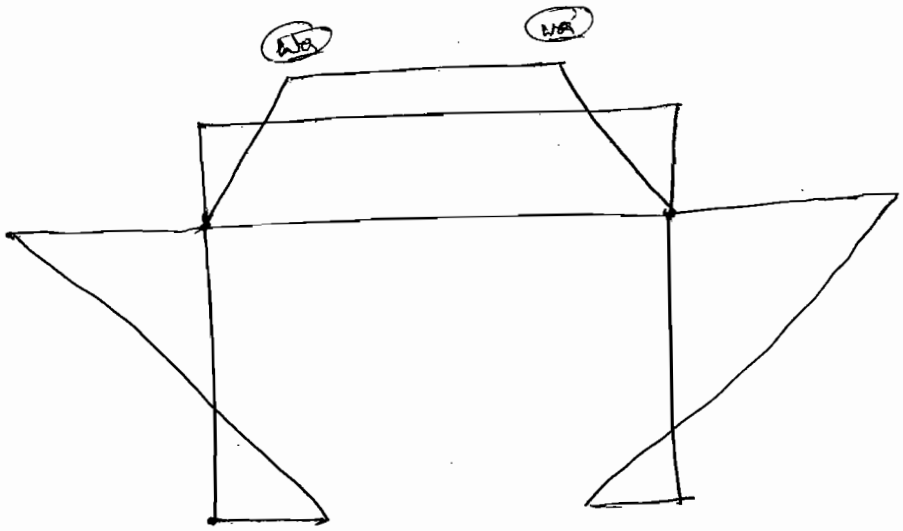
$M_{BC} = -45 + 2(22.46) + (-22.46) = -22.54$ kN-m

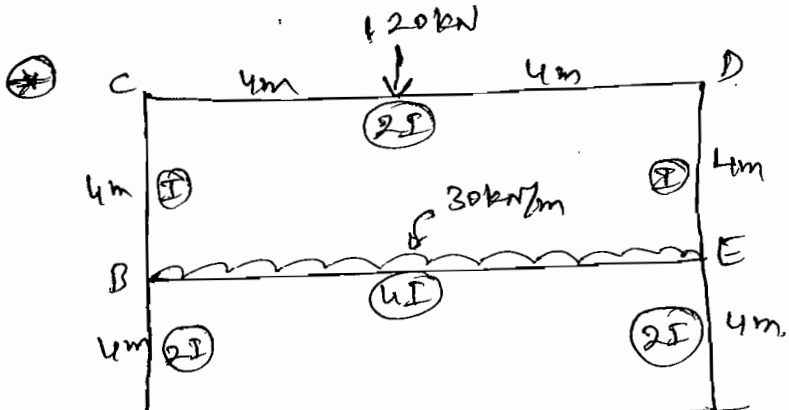
$M_{CB} = 45 + 2(-22.46) + 22.46 = 22.54$ kN-m

$M_{CD} = 0 + 2(-11.26) + 0 = -22.52$ kN-m

$M_{DC} = 0 + 2(0) - 11.26 = -11.26$ kN-m







$M_{AB} = 30.19$

$M_{BA} = 60.38$

$M_{BC} = 66.90$

$M_{BE} = -127.30$

$M_{CD} = 84.75$

$M_{DC} = -84.75$

Stiffness Matrix Method.

The systematic development of slope-deflection method in the matrix form ~~and~~ has lead to stiffness matrix method. This method is also called as displacement method. Since the basic unknowns are the displacement at the joints.

Slope deflection Method.

$$M_{AB} = \frac{2EI}{L}\left(2\theta_A + \theta_B - \frac{3\delta}{L}\right) + M_{FAB}.$$

$$M_{BA} = M_{FBA} + \frac{2EI}{L}\left(2\theta_B + \theta_A - \frac{3\delta}{L}\right)$$

Additional moment due to "Rotation"

At near end $= \dfrac{4EI\theta}{L}$.

At far end $= \dfrac{2EI\theta}{L}$

Due to Sinking

$$\boxed{m = -\frac{6EI\Delta}{L^2}}$$

~~The~~ Stiffness Matrix Method.

$$[R][K] = [P] - [P_L]$$

$$\therefore [R] = [\Delta]$$

$$\boxed{[R] = [K]^{-1}\{[P] - [P_L]\}}$$

where, $[R] = [\Delta]$ = Redundants (Slope 'θ' are taken as unknowns).

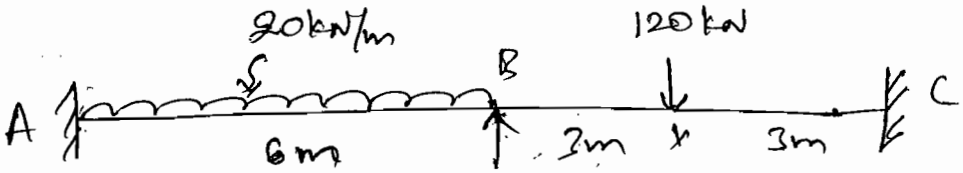
$[K]$ = Stiffness Matrix.

$[P]$ = loads due to Overhang.

$[P_L]$ = Net moment at redundant points (joints).

Problems.

1) Analyse the beam shown in figure by S.M method, draw BMD & EC.



Sol:

1) Degree of Redundancy.

Support 'B' is Simply supported (or might be Hinged/Roller)

∴ θ_B is taken as redundant.

∴ One degree of redundant beam.

2) FEM..

$$M_{FAB} = -\frac{Wl^2}{12} = -60\ kN\text{-}m$$

$$M_{FBA} = \frac{Wl^2}{12} = 60\ kN\text{-}m$$

$$M_{FBC} = -\frac{Wl}{8} = -\frac{120\times 6}{8} = -90\ kN\text{-}m$$

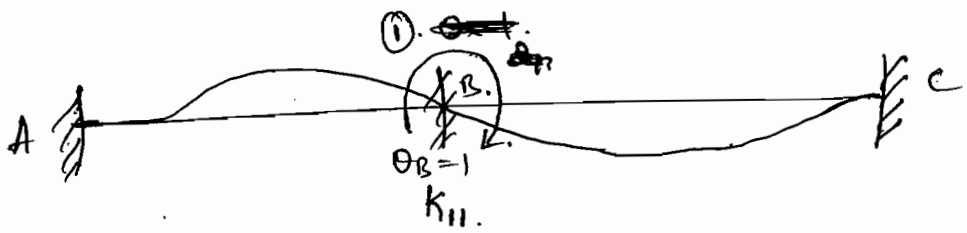
$$M_{FCB} = \frac{Wl}{8} = 90\ kN\text{-}m$$

∴ Net moments at joint are.

@ B. $[P_L] = [M_{FBA} + M_{FBC}] = [60 - 90] = [-30]$.

③ Stiffness Matrix.

Remove all external loads & apply [Clockwise] unit rotation (1) at B.



$\theta_A = 0$
$\theta_C = 0$
$\Delta = 0$.

$$[K] = \frac{2EI}{L}\left(2\theta_B + \cancel{\theta_A} - \cancel{\frac{3\delta}{L}}\right) + \frac{2EI}{L}\left(2\theta_B + \cancel{\theta_C} - \cancel{\frac{3\delta}{L}}\right)$$

$$= \frac{4EI}{6} + \frac{4EI}{6}.$$

$$[K] = 1.33EI.$$

Using S.M equation

$$[R] = [K]^{-1}\{[P] - [P_L]\}.$$

$$\theta_B = [1.33EI]^{-1}\{[0 - (-30)]\}.$$

$$= \frac{1}{1.33EI}(30).$$

$$\boxed{\theta_B = \frac{22.55}{EI}.}$$

(4) Final Moments.

$$M_{AB} = M_{FAB} + \frac{2EI}{6}\left(2\cancel{\theta_A}^{\,0} + \theta_B - \cancel{\frac{3\delta}{L}}^{\,0}\right)$$

$$= -60 + \frac{2EI}{6}\left(\frac{22.55}{EI}\right)$$

$$M_{AB} = -52.48 \text{ kN-m.}$$

$$M_{BA} = 60 + \frac{2EI}{6}(2\theta_B)$$

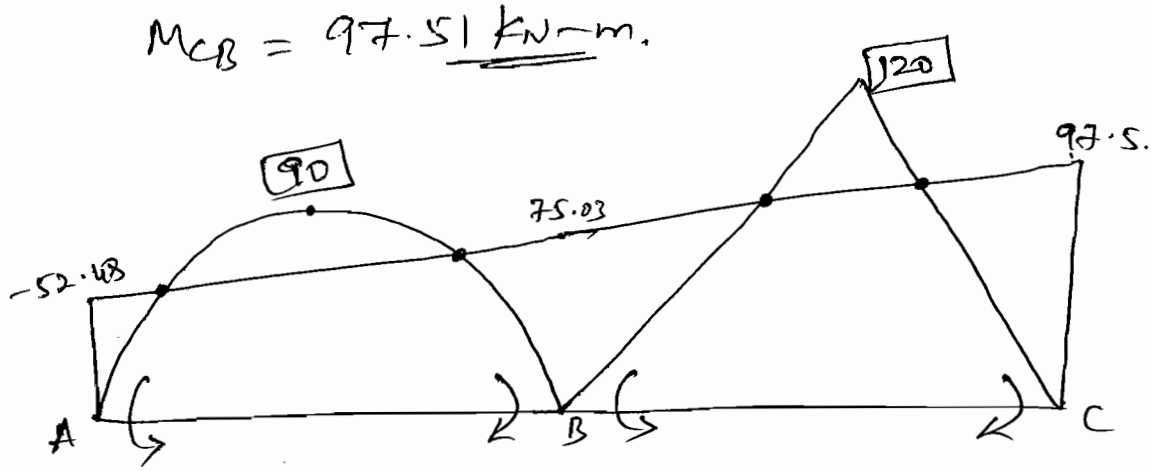
$$= 75.03 \text{ kN-m.}$$

$$M_{BC} = -90 + \frac{2EI}{6}(2\theta_B)$$

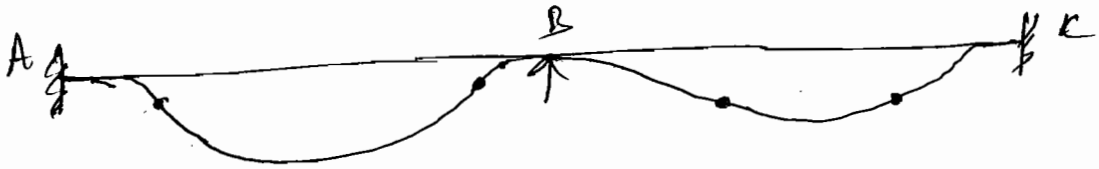
$$= -75 \text{ kN-m.}$$

$$M_{CB} = 90 + \frac{2EI}{6}(\theta_B)$$

$$M_{CB} = 97.51 \text{ kN-m.}$$



BMD.



EC.

② Analyse the C.B shown in figure, draw BMD & C.

$\theta_B = \frac{-11.875}{EI}$

$\theta_C = -\frac{22.188}{EI}$.

Sol:

① Degree of Redundancy.

Support B & C are roller.
① ②

∴ θ_B & θ_C are redundant.

∴ Two degree of redundant beam.

② FEM.

$M_{FAD} = -\frac{60 \times 4^2}{12} = -80 \text{ kN-m}$

$M_{FBA} = 80 \text{ kN-m}$

$M_{FBC} = -\frac{100 \times 3}{8} = -37.5 \text{ kN-m}$

$M_{FCB} = +37.5 \text{ kN-m}$

∴ Net moments at joints are

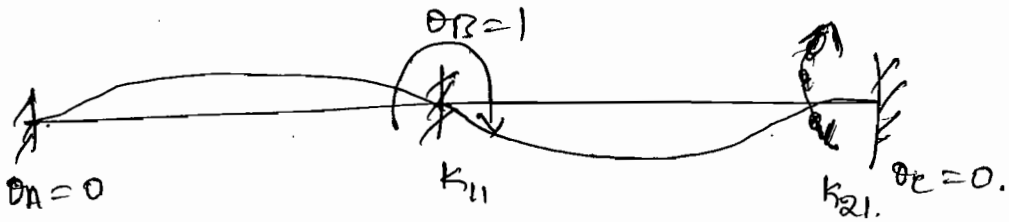
@ joint B. = $M_{FBA} + M_{FBC} = 80 - 37.5$
$= 42.5 \text{ kN-m}$

@ joint C = $M_{FCB} = 37.5 \text{ kN-m}$

$$\therefore [P_L] = \begin{bmatrix} 42.5 \\ 37.5 \end{bmatrix}$$

(3) Stiffness matrix

(*) Unit rotation @ B. ($\theta_B = 1$)



$$k_{11} = M_{BA} + M_{BC}.$$

$$= \frac{2EI}{4}(2\theta_B) + \frac{2EI}{3}(2\theta_B)$$

$$= EI\theta_B + 1.33EI.$$

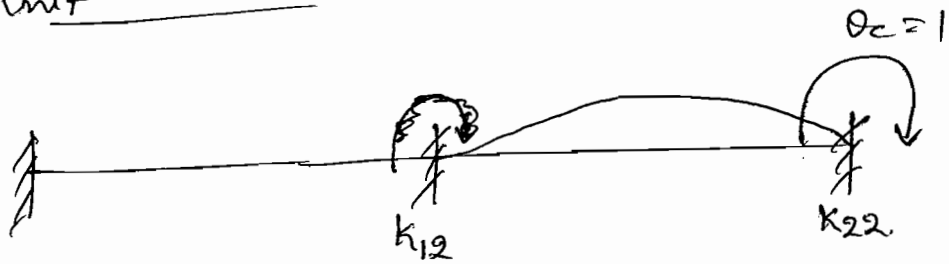
$$= 2.33EI.$$

$$k_{21} = M_{CB}.$$

$$= \frac{2EI}{3}(\theta_B)$$

$$= 0.66EI.$$

(*) Unit rotation @ C. ($\theta_C = 1$)



$$k_{12} = M_{BC}$$

$$= \frac{2EI}{3}(\theta_C)$$

$$= 0.66EI.$$

$$k_{22} = M_{CB}.$$

$$= \frac{2EI}{3}(2\theta_C)$$

$$= 1.33EI.$$

$$K = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} = \begin{bmatrix} 2.33EI & 0.66EI \\ 0.66EI & 1.33EI \end{bmatrix}$$

Using S.M Equ's

$$[R] = [K]^{-1}\{[P]-[P_L]\}$$

$$\begin{bmatrix} \theta_B \\ \theta_C \end{bmatrix} = \begin{bmatrix} 2.33EI & 0.66EI \\ 0.66EI & 1.33EI \end{bmatrix}^{-1} \begin{bmatrix} 0 - 42.5 \\ 0 - 37.5 \end{bmatrix}$$

$$\begin{bmatrix} \theta_B \\ \theta_C \end{bmatrix} = \frac{1}{EI}\begin{bmatrix} -11.93 \\ -22.2 \end{bmatrix}$$

(4) Final Moments.

$$M_{AB} = -80 + \frac{2EI}{4}\left(-11.93/EI\right)$$

$$= -85.96 \text{ kN-m}$$

$$M_{BA} = 80 + \frac{2EI}{4}\left(2\times -11.93/EI\right)$$

$$= 68.07 \text{ kN-m}$$

$$M_{BC} = -37.5 + \frac{2EI}{3}\left(2\times -11.93/EI + \frac{-22.2}{EI}\right)$$

$$= -68.2 \text{ kN-m}$$

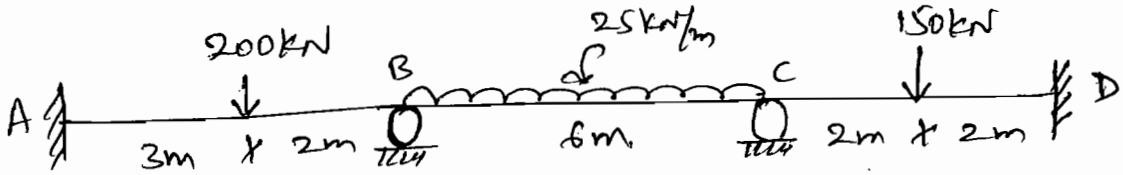
$$M_{CB} = +37.5 + \frac{2EI}{3}\left[\frac{-11.93}{EI} + 2\times\frac{-22.2}{EI}\right]$$

$$= 0.$$

Inverse of Matrix

$$[K]^{-1} = \frac{1}{(2.33\times1.33)-(0.66\times0.66)}\begin{bmatrix} 1.33 & -0.66 \\ -0.66 & 2.33 \end{bmatrix} = \begin{bmatrix} 0.498 & -0.247 \\ -0.247 & 0.873 \end{bmatrix}$$

③ Analyse the C.B shown in fig, Draw B.M.D & E.C.



Sol:-

① D.D.R.

θ_B & θ_C are taken as Redundants.

② FEM.

$M_{FAB} = -96$ kN-m $\qquad M_{FBA} = 144$ kN-m

$M_{FBC} = -75$ kN-m $\qquad M_{FCB} = 75$ kN-m

$M_{FCD} = -75$ kN-m $\qquad M_{FDC} = +75$ kN-m

③ ∴ Net moment @ joints are.

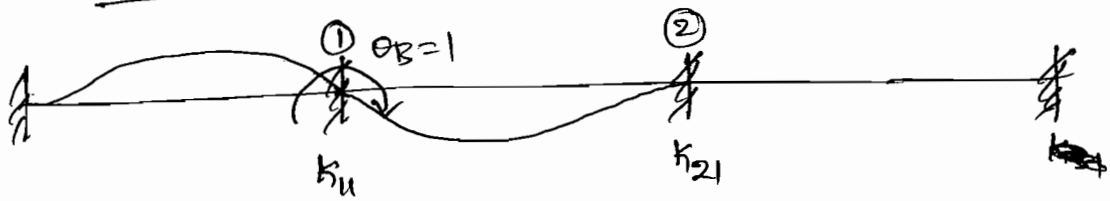
@ B = 144 − 75 = 69 kN-m (P_{L1})

@ C = 75 − 75 = 0 (P_{L2})

$$P_L = \begin{bmatrix} 69 \\ 0 \end{bmatrix}$$

③ Stiffness Matrix.

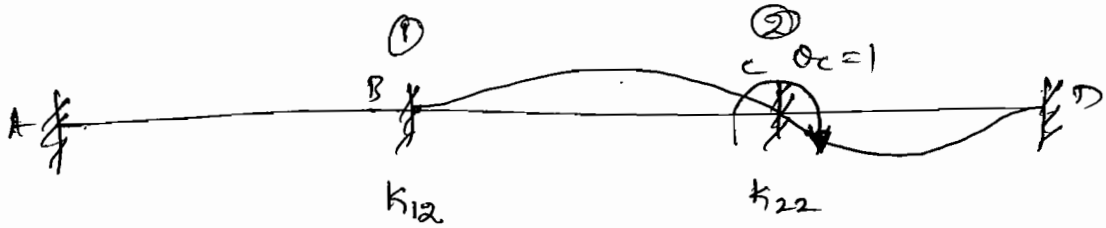
Unit Rotation @ B



$$K_{11} = \frac{2EI}{5}(2\theta_B) + \frac{2EI}{6}(2\theta_B) = 1.467EI.$$

$$K_{21} = \frac{2EI}{6}(\theta_B) = 0.33\,EI.$$

(*) Unit Rotation @ C. ($\theta_C = 1$)



$$k_{12} = \frac{2EI}{6}(\theta_C) = 0.33\,EI.$$

$$k_{22} = \frac{2EI}{6}(2\theta_C) + \frac{2EI}{4}(2\theta_C) = 1.67\,EI.$$

$$[k] = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} = \begin{bmatrix} 1.467 & 0.33 \\ 0.33 & 1.67 \end{bmatrix} EI.$$

Using S.M. Eqn's

$$[R] = [k]^{-1}\{[P] - [P_L]\}.$$

$$\begin{bmatrix} \theta_B \\ \theta_C \end{bmatrix} = \begin{bmatrix} 1.467 & 0.33 \\ 0.33 & 1.67 \end{bmatrix}^{-1} \begin{bmatrix} 0 - 69 \\ 0 - 0 \end{bmatrix}$$

$$\begin{bmatrix} \theta_B \\ \theta_C \end{bmatrix} = \begin{bmatrix} -49.22/EI \\ 9.72/EI \end{bmatrix}$$

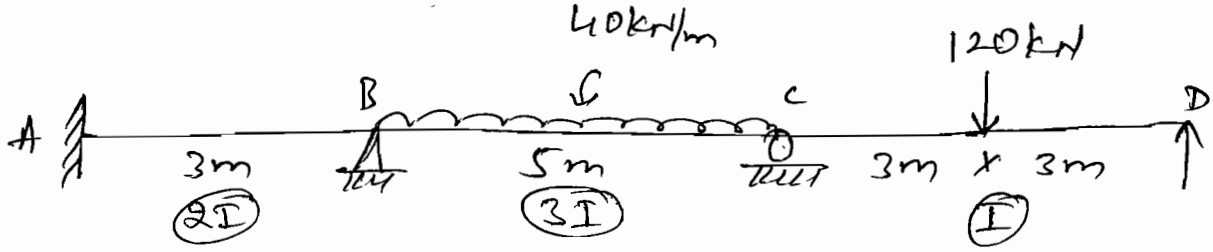
(4) Final Moments.

$M_{AB} = -115.71$ kN-m $\qquad M_{BA} = 104.58$ kN-m

$M_{BC} = -104.58$ kN-m $\qquad M_{CB} = 65.15$ kN-m

$M_{CD} = -65.14$ kN-m $\qquad M_{DC} = 79.92$ kN-m

(4) Analyse the C.B by Stiffness Matrix method. Draw BMD & EC.



Sol:- (1) D.O.R.

The supports B, C, & D are ~~SS~~ Hinged, roller & ~~Hinged~~ SS.

$\therefore$ θ_B, θ_C & θ_D are redundants.

(2) FEM.

$M_{FAB} = M_{FBA} = 0.$

$M_{FBC} = -\frac{Wl^2}{12} = -83.33\,kN\text{-}m$

$M_{FCB} = 83.33\,kN\text{-}m$

$M_{FCD} = -90\,kN\text{-}m$ $\quad M_{FDC} = 90\,kN\text{-}m$

Net moments

@ 'B', $P_{L1} = M_{FBA} + M_{FBC} = -83.33\,kN\text{-}m$

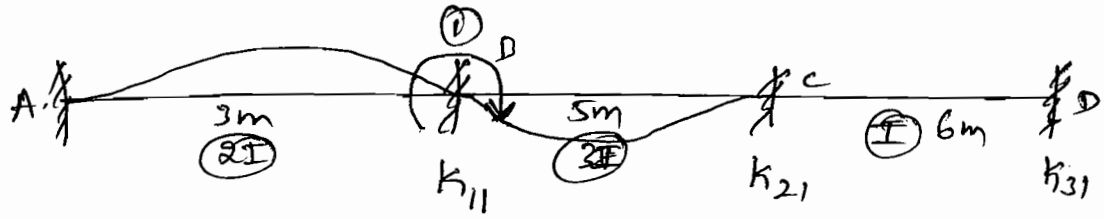
@ 'C', $P_{L2} = M_{FCB} + M_{FCD} = 83.33 - 90 = -6.67\,kN\text{-}m$

@ 'D' $P_{L3} = M_{FDC} = 90\,kN\text{-}m$

$$\therefore [P_L] = \begin{bmatrix} -83.33 \\ -6.67 \\ 90 \end{bmatrix} \qquad P = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$ $\because$ No overhang

(3) Stiffness Matrix Method

(*) Unit rotation @ B. ($\theta_B = 1$)



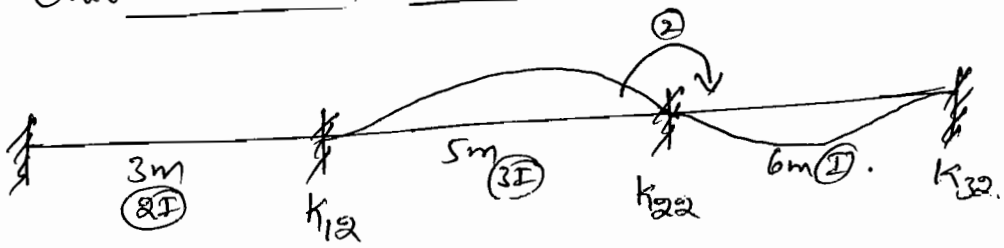
$$K_{11} = \frac{2E(2I)}{3}(2\theta_B) + \frac{2E(3I)}{5}(2\theta_B)$$

$$= 5.06\,EI.$$

$$K_{21} = \frac{2E(3I)}{5}(\theta_B) = 1.2\,EI.$$

$$K_{31} = 0.$$

(*) Unit rotation @ C ($\theta_C = 1$)



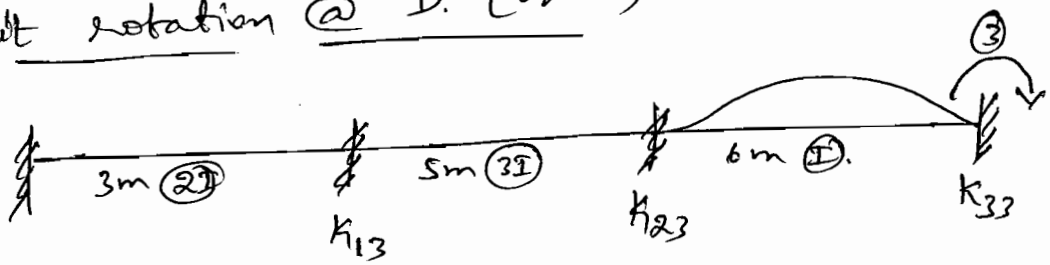
$$K_{12} = \frac{2E(3I)}{5}(\theta_C) = 1.2\,EI.$$

$$K_{22} = \frac{2E(3I)}{5}(2\theta_C) + \frac{2E(I)}{6}(2\theta_C)$$

$$= 3.067\,EI.$$

$$K_{32} = \frac{2E(I)}{6}(\theta_C) = 0.333\,EI.$$

(*) Unit rotation @ D. ($\theta_D = 1$)



$$K_{13} = 0.$$

$$k_{23} = \frac{2E(I)}{6}(\theta_D) = 0.333EI.$$

$$k_{33} = \frac{2E(I)}{6}(2\theta_D) = 0.666EI.$$

$$[k] = \begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \\ k_{31} & k_{32} & k_{33} \end{bmatrix} = \begin{bmatrix} 5.06 & 1.2 & 0 \\ 1.2 & 3.06 & 0.333 \\ 0 & 0.333 & 0.666 \end{bmatrix} EI.$$

$$[R] = [k]^{-1}[P - P_L]$$

$$\begin{bmatrix} \theta_B \\ \theta_C \\ \theta_D \end{bmatrix} = EI \begin{bmatrix} 5.06 & 1.2 & 0 \\ 1.2 & 3.06 & 0.333 \\ 0 & 0.333 & 0.666 \end{bmatrix}^{-1} \begin{bmatrix} 0 - (-83.33) \\ 0 - (-6.67) \\ 0 - (90) \end{bmatrix}$$

$$\boxed{\begin{aligned} \theta_B &= 13.56/EI. \\ \theta_C &= 12.23/EI. \\ \theta_D &= -141.25/EI. \end{aligned}}$$

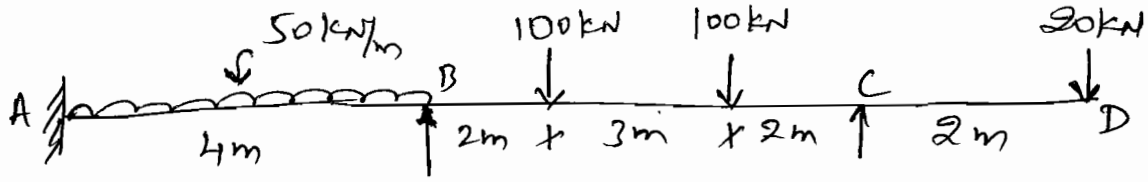
④ Final Moments.

$M_{AB} = 18.13$ kN-m $\qquad M_{BA} = 36.27$ kN-m

$M_{BC} = -36.08$ kN-m $\qquad M_{CB} = 128.90$ kN-m

$M_{CD} = -128.9$ kN-m $\qquad M_{DC} = 0.$

(b) Analyse the C.B shown in fig, using D.M.M.



Sol:-

① D.O.R.

θ_B & θ_C are Redundants.

② F.E.M.

$M_{FAB} = -66.67\,kN\text{-}m$ $\quad M_{FBA} = +66.67\,kN\text{-}m$

$M_{FBC} = -142.85\,kN\text{-}m$ $\quad M_{FCB} = 142.85\,kN\text{-}m$

Convert overhang in actual moment.

$\therefore M_{CD} = 20 \times 2 = +40\,kN\text{-}m$ ↻

[Clockwise +ve]

$$\therefore \quad P = \begin{bmatrix} 0 \\ 40 \end{bmatrix}$$

Net moment @ joints

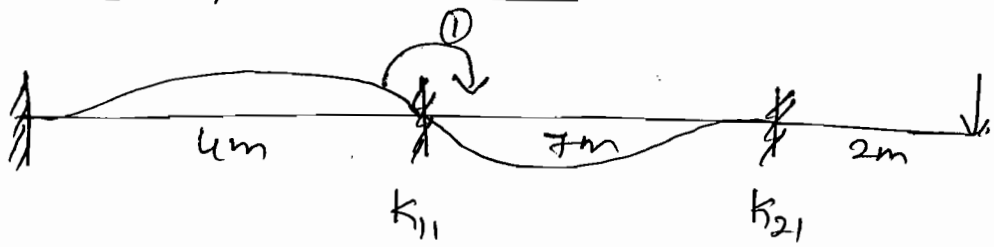
@ B, $M_{FBA} + M_{FBC} = +66.67 - 142.85 = -76.18\,kN\text{-}m$

@ C, $M_{FCD} = 142.85\,kN\text{-}m$

$$[P_L] = \begin{bmatrix} -76.18 \\ 142.85 \end{bmatrix}$$

(3) S.M.

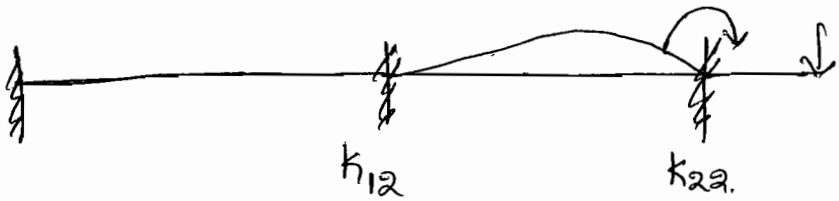
(*) Unit displacement @ B. ($\theta_B = 1$)



$$k_{11} = \frac{2EI}{4}(2\theta_B) + \frac{2EI}{7}(2\theta_B) = 1.57EI$$

$$k_{21} = \frac{2EI}{7}(\theta_B) = 0.285EI$$

(*)



$$k_{12} = \frac{2EI}{7}(\theta_C) = 0.285EI$$

$$k_{22} = \frac{2EI}{7}(2\theta_C) = 0.57EI$$

$$[K] = \begin{bmatrix} 1.57 & 0.285 \\ 0.285 & 0.57 \end{bmatrix} EI.$$

$$[R] = [K]^{-1}[P - P_L]$$

$$\begin{bmatrix} \theta_B \\ \theta_C \end{bmatrix} = EI \begin{bmatrix} 1.57 & 0.285 \\ 0.285 & 0.57 \end{bmatrix}^{-1} \begin{bmatrix} 0 - (-76.18) \\ 40 - (142.85) \end{bmatrix}$$

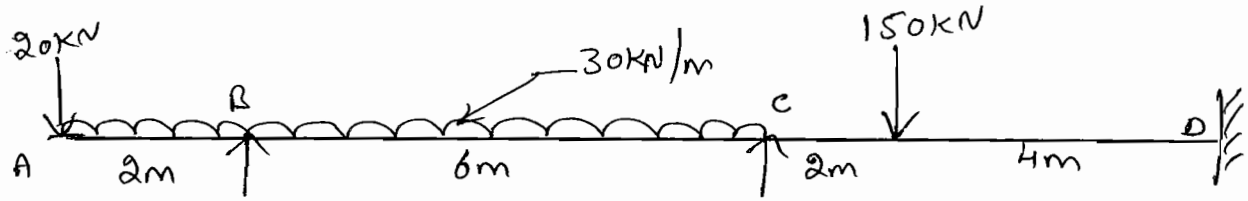
$$\boxed{\theta_B = 89.39/EI \qquad \theta_C = -225.13/EI.}$$

(4) Final Moments.

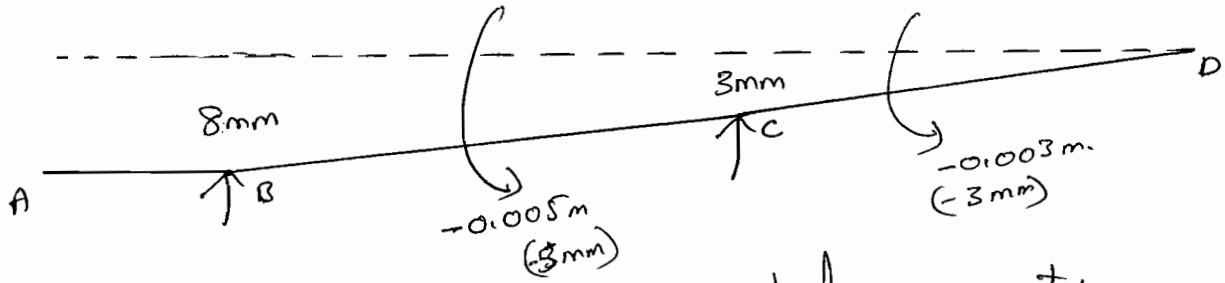
$M_{AB} = -21.97$ kN-m $\qquad M_{BA} = 156.06$ kN-m

$M_{BC} = -156.09$ kN-m $\qquad M_{CB} = +40$ kN-m

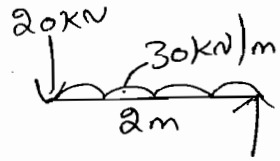
6) Analyse the beam Shown by S.M method. The Support 'B' & 'C' Sinks by 8mm & 3mm. Take $EI = 8000$ kN-m^2.



Soln



Converting overhang into actual moment.



$\therefore M_B = -20\times 2 - 30\times 2\times \frac{2}{2} = \underline{\underline{-100 \text{ kN-m}}}$

-ve

$$[P] = \begin{bmatrix} -100 \\ 0 \end{bmatrix}$$

a) Degree of Redundancy

'θ_B' & θ_C are Redundants.

$\therefore$ Two degree of Redundant beam.

b) FEM's

$$M_{FBC} = \frac{-wl^2}{12} - \frac{6EI\delta}{l^2} = \frac{-30\times 6^2}{12} - \frac{6\times 8000\times(-0.005)}{6^2}$$

$$= \underline{\underline{-83.33}} \text{ kN-m.}$$

$$M_{FCB} = \frac{Wl^2}{12} - \frac{6EI\delta}{l^2} = 96.67 \text{ kN-m}$$

$$M_{FCD} = \frac{-Wab^2}{l^2} - \frac{6EI\delta}{l^2} = \frac{-150 \times 2 \times 4^2}{6^2} - \frac{6(8000)(-0.003)}{6^2}$$

$$= -129.33 \text{ kN-m}$$

$$M_{FDC} = \frac{Wa^2b}{l^2} - \frac{6EI\delta}{l^2} = 70.67 \text{ kN-m}$$

∴ Net moments

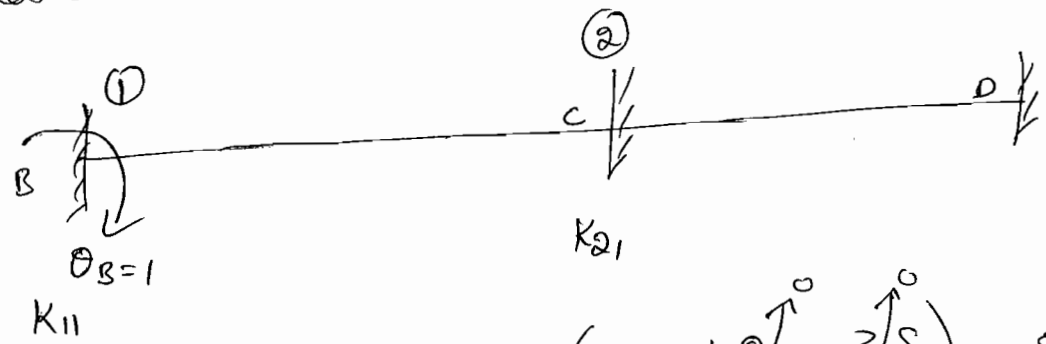
at 'B' ⇒ $M_{BC} = -83.33$ kN-m

at 'C' ⇒ $M_{FCB} + M_{FCD} = -32.66$ kN-m

$$[P_2] = \begin{bmatrix} -83.33 \\ -32.66 \end{bmatrix}$$

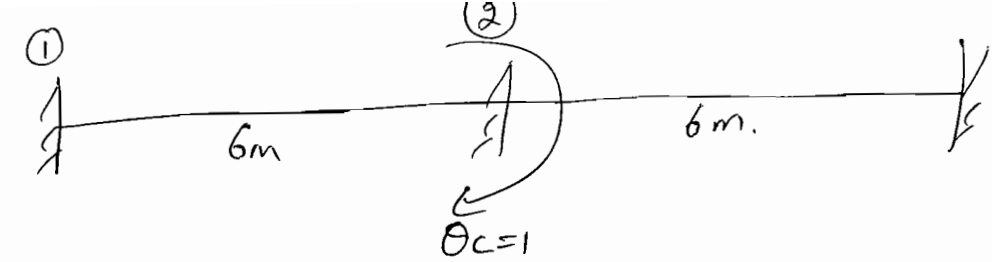
(c) Stiffness matrix

Remove all the external loads & apply unit rotation at 'B' & 'C'.



$$K_{11} = \cancel{\frac{4EI}{l}} = \frac{2EI}{6}\left(2\theta_B + \cancel{\theta_C}^{\,0} - \cancel{\frac{3\delta}{l}}^{\,0}\right) = 5333.33$$

$$K_{21} = \frac{2EI}{6}\left(2\cancel{\theta_C}^{\,0} + \theta_B - \cancel{\frac{3\delta}{l}}^{\,0}\right) = 2666.67$$



K12 K22

$$K_{12} = \frac{2EI}{6}\left(2\cancel{\theta_B}^{\,0} + \theta_c - \cancel{\frac{3\delta}{l}}^{\,0}\right) = 2666.67$$

$$K_{22} = \frac{2EI}{6}\left(2\theta_c + \cancel{\theta_B}^{\,0} - \cancel{\frac{3\delta}{l}}^{\,0}\right) + \frac{2EI}{6}\left(2\theta_c + \cancel{\theta_B}^{\,0} - \cancel{\frac{3\delta}{l}}^{\,0}\right)$$

$$K_{22} = 10666.66$$

$$[K] = \begin{bmatrix} 5333.33 & 2666.67 \\ 2666.67 & 10666.67 \end{bmatrix}$$

$$[R] = [K]^{-1}\left\{[P] - [P_L]\right\}$$

$$\begin{bmatrix} \theta_B \\ \theta_c \end{bmatrix} = \begin{bmatrix} 5333.33 & 2666.67 \\ 2666.67 & 10666.67 \end{bmatrix}^{-1} \left\{ \begin{bmatrix} -100 \\ 0 \end{bmatrix} - \begin{bmatrix} -83.33 \\ -32.66 \end{bmatrix} \right\}$$

$$\begin{bmatrix} \theta_B \\ \theta_c \end{bmatrix} = \begin{bmatrix} -5.322 \times 10^{-3} \\ 4.392 \times 10^{-3} \end{bmatrix}$$

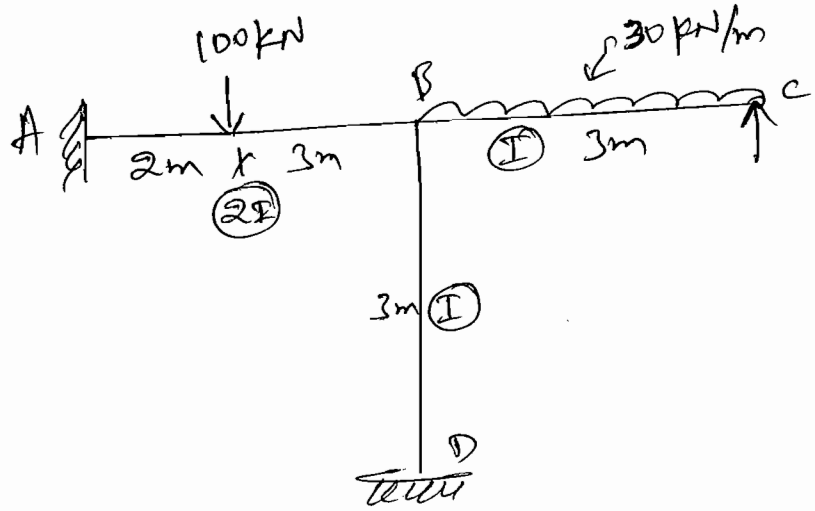
Final moments.

$M_{BC} = -100 kN-m$, $M_{CB} = 105.9 kN-m.$

$M_{CD} = -105.9 kN-m$, $M_{DC} = 82.38 kN-m.$

Analysis of Non-Sway Frames.

① Analyse the frame shown in figure.



Sol

ⓐ D.O.R

θ_B, θ_C are Redundants (S.S)

∴ 2 degree of redundant frame

ⓑ F.E.M.

$M_{FAB} = -\frac{Wab^2}{l^2} = -72 kN-m$

$M_{FBA} = -\frac{Wa^2b}{l^2} = 48 kN-m$

$M_{FBC} = -\frac{Wl^2}{12} = -22.5 kN-m$

$M_{FCB} = \frac{Wl^2}{12} = 22.5 kN-m$

$M_{FCD} = M_{FDC} = 0$

Net moments @ redundant joints.

@ 'B', = ~~[illegible]~~ 48 − 22.5 = 25.5 kN-m

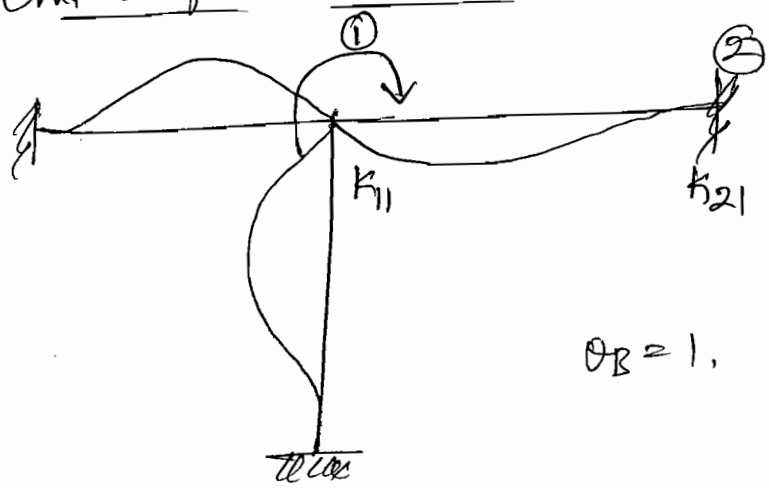
@ 'C', = 22.5 kN-m

$$P_L = \begin{bmatrix} 25.5 \\ 22.5 \end{bmatrix}$$

$$P = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (\because \text{No overhang})$$

③ S.M.

(*) Unit displacement @ B.



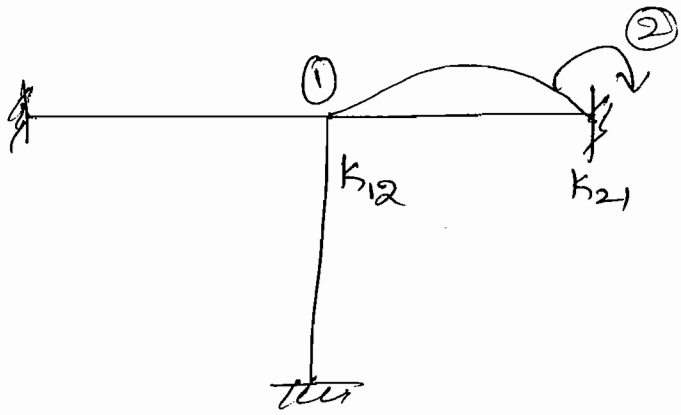
$\theta_B = 1.$

$$K_{11} = \frac{2E(2I)}{5}(2\theta_B) + \frac{2EI}{3}(2\theta_B) + \frac{2EI}{3}(2\theta_B)$$

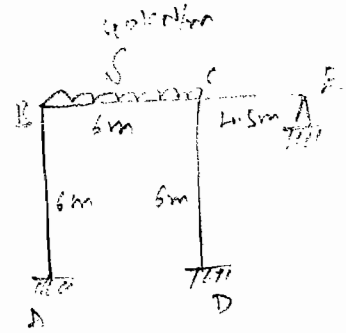
$$K_{11} = 4.267\,EI.$$

$$K_{21} = \frac{2EI}{3}(\theta_B) = 0.667\,EI.$$

(*) Unit rotation @ C [$\theta_C = 1$]



Problem - 2



$$k_{12} = \frac{2EI}{3}(\theta_C) = 0.667EI$$

$$k_{22} = \frac{2EI}{3}(2\theta_C) = 1.33\,EI$$

$$[k] = \begin{bmatrix} 4.267 & 0.667 \\ 0.667 & 1.333 \end{bmatrix} EI$$

$$[R] = [k]^{-1}[P - P_L]$$

$$\begin{bmatrix} \theta_B \\ \theta_C \end{bmatrix} = \frac{1}{EI}\begin{bmatrix} 4.267 & 0.667 \\ 0.667 & 1.333 \end{bmatrix}^{-1}\begin{bmatrix} 0 - 25.5 \\ 0 - 22.5 \end{bmatrix}$$

$$\boxed{\begin{aligned} \theta_B &= -3.62/EI \\ \theta_C &= -15.06/EI \end{aligned}}$$

(4) Final moments.

$M_{AB} = -74.88$ kN-m

$M_{BC} = -37.37$ kN-m

$M_{BD} = -4.81$ kN-m

$M_{BA} = 42.23$ kN-m

$M_{CB} = \cancel{-4.81}\ 0.$

$M_{DB} = -2.402$ kN-m